1. You are given the potentials $\vec{A} = xz\cos\omega t\hat{x}$, $\phi = 0$.

a. Find the electric field $\vec{E}$ and magnetic field $\vec{B}$.

b. Find the gauge equivalent potentials $(\phi', \vec{A}')$ in Coulomb gauge, $\nabla \cdot A' = 0$, such that A' lies along the $\hat{z}$ axis.

c. Evaluate $\oint_C \vec{B} \cdot d\vec{l}$ for $C = C_{xy}$, $C = C_{xz}$, and $C = C_{yz}$, with C_{xy} a circle of radius R, centered at the origin and lying in the x-y plane (likewise for C_{xz} and C_{yz}). Feel free to use Stokes theorem.

d. What is the charge density $\rho(\vec{x}, t)$ and current density $\vec{J}(\vec{x}, t)$?

e. Consider a unit cube with the following six faces: (face 1:) ($x = 0$, $0 \leq y \leq 1$, $0 \leq z \leq 1$); (face 2:) ($y = 0$, $0 \leq x \leq 1$, $0 \leq z \leq 1$); (edge 3:) ($z = 0$, $0 \leq x \leq 1$, $0 \leq y \leq 1$); (face 4:) ($x = 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$); (face 5:) ($y = 1$, $0 \leq x \leq 1$, $0 \leq z \leq 1$); (face 6:) ($z = 1$, $0 \leq x \leq 1$, $0 \leq y \leq 1$). Find the total charge $Q(t)$ inside this cube as a function of time. Find the total current $I(t)_n$ going *out* of the cube (put in a minus sign if the current is going *into the cube*) through each of the above six faces ($n = 1\ldots6$), as a function of time. Check that these six currents add up to $\frac{-dQ(t)}{dt}$, the amount by which the charge inside the cube decreases, as is required by charge conservation.

2. A "dyon" is a (hypothetical) particle with both electric charge q and magnetic charge g. The force on a dyon in electric and magnetic fields is

$$\vec{F} = q\vec{E} + \frac{q}{c}\vec{v} \times \vec{B} + g\vec{B} - \frac{g}{c}\vec{v} \times \vec{E},$$

where the contributions proportional to g follow from the first two with the replacements $q \to g$, $\vec{E} \to \vec{B}$, $\vec{B} \to -\vec{E}$ mentioned in lecture.

a. Find the electric and magnetic fields due to a dyon with electric charge q_1 and magnetic charge g_1 which is nailed down at the origin (it doesn't move).

b. Find the force on a second dyon, with charges q_2 and g_2, at position $\vec{r}$, and velocity $\vec{v}$ (nonrelativistic), in the background of the dyon at the origin of part (a.). Is the angular momentum of the second dyon about the origin conserved? Is the magnitude of the angular momentum conserved?