Homework 6, due Nov. 29, 2006

1. Problem 11.1 in book. Here is a nice way to do the integrals: note that $\int_0^\infty e^{-\lambda v} dv = \lambda^{-1}$ (for $\lambda > 0$), and then take $(-1)^n d^n/d\lambda^n$ of both sides to get $\int_0^\infty v^n e^{-\lambda v} dv = (-1)^n d^n(\lambda)^{-1}/d\lambda^n = n!\lambda^{-n-1}$.

2. Problem 11.2 in book.

3. Problem 11-15 in book.

4. Consider a gas of one kilomole of He atoms, at $T = 300K$ and $P = 1atm$. The mean energy is $\bar{\epsilon} = \frac{3}{2}kT \approx 6 \times 10^{-21} J$. Estimate the number of atoms of this gas whose energy ϵ lies in an interval of width $10^{-22} J$ around this mean value. Hint: use the Maxwell distribution. It's a bit simpler to write it in terms of ϵ for this problem.

5. Suppose you flip 1000 unbiased coins.

   (a) What is the probability of getting 500 heads and 500 tails? (Use Stirling's approximation.)

   (b) What is the probability of getting 450 heads and 550 tails? (Use Stirling again.)

   (c) Compute the above two probabilities using the Gaussian approximation. How do they compare?

   (d) Compute $\ln\Omega$ and $\ln\omega_{max}$. How do they compare?

6. A drunk person is walking along the x axis. He starts at $x = 0$, and his step size is $L = 0.5$ meters. For each step, he has chance 2/3 of walking forwards (positive x) and chance 1/3 of walking backwards (negative x). What is his expected position, $\overline{x}$, after 50 steps? What is the expected RMS variation around $\overline{x}$, $\Delta x_{RMS} = \sqrt{\overline{(x - \overline{x})^2}}$ after 50 steps?

7. Suppose that a system has allowed energy levels $n\epsilon$, with $n = 0, 1, 2, 3, 4, \ldots$. There are three distinguishable particles, with total energy $U = 4\epsilon$.

   (a) Tabulate all possible distributions of the three particles among the energy levels, satisfying $U = 4\epsilon$.

   (b) Evaluate ω_i for each of above distributions, and also $\Omega = \sum_i \omega_i$.

   (c) Calculate the average occupation numbers $\overline{N}_n = \sum_k N_{nk}\omega_k/\Omega$ for the three particles in the energy states. Here N_n is the average occupation number of the energy level with energy $n\epsilon$. You should find N_n for all $n \leq 4$ (and find that $N_{n>4} = 0$).