11/18 Lecture outline

⋆ **Reading: Luke chapter 10. Tong chapter 5**

• Hamiltonian of the Dirac equation, with fermionic statistics, $\mathcal{H} = \pi_\psi \dot{\psi} - \mathcal{L} = \bar{\psi}(-i\partial_j \gamma^j + m)\psi$, and then $H = \int d^3x \mathcal{H}$ gives

$$:H: = \int \frac{d^3p}{(2\pi)^3} E_p (b^{r\dagger}(p) b^r(p) + c^{r\dagger}(p) c^r(p)),$$

good, $b^{r\dagger}(p)$ creates a spin 1/2 particle of positive energy, and $c^{r\dagger}(p)$ creates a spin 1/2 particle of positive energy (the second term would have had the wrong sign with bosonic statistics).

• Do perturbation theory as before, but account for Fermi statistics, e.g. $T(\psi(x_1)\psi(x_2)) = -T(\psi(x_2)\psi(x_1))$ and likewise for normal ordered products. Consider in particular the propagator

$$\{\psi(x), \bar{\psi}(y)\} = (i\partial\!\!\!/_x + m)(D(x-y) - D(y-x)).$$

and the contraction

$$\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle = \int \frac{d^4p}{(2\pi)^4} \frac{i(p\!\!\!/ + m)}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}.$$

Vanishes for spacelike separated points. The momentum space fermion propagator is

$$\frac{i}{p\!\!\!/ - m + i\epsilon}.$$

Let's call the particle states nucleons and anti-nucleons (we could also call them electrons and positrons etc):

$$|N(p,r)\rangle = b(p)^{r\dagger}|0\rangle \qquad |\bar{N}(p,r)\rangle = c^{r\dagger}(p)|0\rangle.$$

Then

$$\langle 0|\psi(x)|N(p,r)\rangle = e^{-ipx}u^r(p), \qquad \langle N(p,r)|\bar{\psi}(x)|0\rangle = e^{ipx}\bar{u}^r(p).$$

Incoming fermions get a factor of $u^r(p)$, outgoing fermions gt $\bar{u}^r(p)$; incoming antifermions gets $\bar{v}^r(p)$, and outgoing antifermions get $v^r(p)$.

Write the amplitude by following the arrows backwards, from the head to the tail.

• Example, redo our meson-nucleon toy model, but now treating the nucleons as fermions, interacting with the scalar via $\mathcal{L}_{int} = -g\phi\bar{\psi}_a \Gamma_{ab} \psi_b(x)$. Compute various scattering amplitudes.

$N + \phi \to N + \phi$:

$$i\mathcal{A} = (-ig)^2 \bar{u}^{r'}(p')\Gamma \left(\frac{i(p\!\!\!/ + q\!\!\!/ + m)}{(p+q)^2 - m^2 + i\epsilon} + \frac{i(p\!\!\!/ - q\!\!\!/' + m)}{(p-q')^2 - m^2 + i\epsilon} \right) \Gamma u^r(p).$$