215a Homework exercises 3, due Nov. 5

Homework exercise key: "Luke problem $n.m$" refers to exercise set n, problem m. Likewise for Tong. Follow links from website.

1. Consider a real scalar field with ϕ^4 interaction: $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4$.

   (a) What is the mass dimension of $\phi(x)$, and of λ, if the above is the Lagrangian density in D spacetime dimensions? Write it as $[\lambda] = n$ if $\lambda \sim (\text{mass})^n$. Recall $[S] = 0$ and $S = \int d^D x \mathcal{L}$. Your answers to all these questions will depend on D. (This exercise will be useful next quarter, when you'll learn about dimensional regularization: taking $D = 4 - \epsilon$, with $\epsilon \ll 1$, taking the limit $\epsilon \to 0$ only at the end of the day.)

   (b) Write $\phi(x) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}2E_k}(a(k)e^{-ik\cdot x} + a^\dagger(k)e^{ikx})$. We'll say that a and $a^\dagger$ annihilate and create "mesons." Using your result from above, what is the mass dimension $[a(k)]$ and $[a^\dagger(k)]$? As a check, it should give the answer found in class for $D = 4$.

   (c) Define $\langle f|S|i\rangle = i\mathcal{A}_{fi}\delta^D(p_f - p_i)$. Suppose that $|i\rangle$ has n_i mesons and $\langle f|$ has n_f mesons. What is the mass dimension $[|i\rangle]$ and $\langle f|]$ and $[\mathcal{A}_{fi}]$? Again, verify that it reduces to the answer given in class for $D = 4$.

   (d) Find the amplitude (with D general) for the decay $\phi \to \phi + \phi + \phi$, to leading order in λ. Verify that your result is consistent with dimensional analysis and the above results.

2. Tong 4.1. Exercise about the ϕ^4 interaction. Here set $D = 4$.

3. Luke 3.2. Exercise about scalar field theory with source $\rho(x^\mu)$ and the Poisson distribution. This exercise is also in Peskin and Schroeder, exercise 4.1.