12/3 Lecture outline

⋆ **Reading: Luke chapter 11. Tong chapter 6**

• Recap: we have discussed spin 0 and spin 1/2 quantum fields. Now move up to spin 1. (Next quarter, we'll discuss renormalizability, and note there the complications with quantizing fields of spin greater than 1.) Examples with spin 1 include non-fundamental (composite) fields, e.g. spin 1 mesons, and also the fundamental force carriers: the photon, gluons, and $W^{\pm}$ and Z^0. The gluons and $W^{\pm}$ are associated with non-Abelian groups, which we won't discuss this quarter (we'll see if we get to it next quarter).

• Consider a spin 1 quantum field (the $(\frac{1}{2}, \frac{1}{2})$ representation of the Lorentz group), and call it A_μ. The components of A_μ will satisfy something like a KG equation, being massive or massless. We'll start with the massive case first, as a warmup for the massless case. Physically, this could be referring to the Z^μ massive vector bosons of the broken electroweak force.

For the massive vector mesons, write down the general lagrangian:

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu A^\nu \partial_\mu A^\nu + a\partial_\mu A^\mu \partial_\nu A^\nu + bA_\mu A^\mu).$$

The sign is chosen so that the kinetic terms of the spatial components have the right sign. Write the EOM:

$$-\partial^2 A_\nu - a\partial_\nu(\partial \cdot A) + bA_\nu = 0,$$

and note plane wave solutions $A_\mu(x) = \epsilon_\nu e^{-ik\cdot x}$ solves it if $k^2\epsilon_\nu + ak_\nu(k\cdot\epsilon) + b\epsilon_\nu = 0$. The longitudinal solutions have $\epsilon \propto k$ and have mass $\mu_L^2 = -b/(1+a)$. The transverse have mass $\mu_T^2 = -b$. Can eliminate the uninteresting longitudinal solution by taking $a = -1$ and $b \neq 0$, then write Proca lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}\mu^2 A_\mu A^\mu,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Each component A_μ satisfies the KG equation with mass μ. Can choose $\epsilon^{(\pm)} = \frac{1}{\sqrt{2}}(0, 1, \mp i, 0)$ and $\epsilon^{(0)} = (0, 0, 0, 1)$, where the label is the value of J_z of the spin 1 vector. Normalize by $\epsilon^{(r)*}\cdot\epsilon^{(s)} = -\delta^{rs}$ and $\sum_r \epsilon_\mu^{(r)*}\epsilon_\nu^{(r)} = -g_{\mu\nu} + \frac{k_\mu k_\nu}{\mu^2}$.

The conjugate momenta to A_μ are $\pi^0 = \partial\mathcal{L}/\partial\dot{A}_0 = 0$, and $\pi^i = \partial\mathcal{L}/\partial\dot{A}_i = -F^{0i} = E^i$. Then $\mathcal{H} = -\frac{1}{2}(F_{0i}F^{0i} - \frac{1}{2}F_{ij}F^{ij} + \mu^2 A_i A^i - \frac{1}{2}\mu^2 A_0 A^0) \geq 0$.

• Quantize the massive vector:

$$[A_i(t,\vec{x}), F^{j0}(t,\vec{y})] = i\delta_i^j \delta^{(3)}(\vec{x}-\vec{y}).$$

In terms of the plane wave solutions,

$$A_\mu(x) = \sum_{r=1}^{3} \int \frac{d^3k}{(2\pi)^{3/2}(\sqrt{2\omega_k})} \left[a_k^r \epsilon_\mu^r e^{-ikx} + a_k^{\dagger r} \epsilon_\mu^{*r} e^{ikx} \right],$$

and then

$$[a_k^r, a_{k'}^{\dagger s}] = \delta^{rs}\delta^3(\vec{k} - \vec{k}').$$

and

$$: \mathcal{H} := \sum_r \int d^3k \omega_k a_k^{\dagger r} a_k^r.$$

The propagator, the contraction of $A_\mu(x)$ and $A_\nu(y)$, is

$$\langle T A_\mu(x) A_\nu(y) \rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \left[\frac{-i(g_{\mu\nu} - k_\mu k_\nu/\mu^2)}{k^2 - \mu^2 + i\epsilon} \right].$$

So the Feynman rule is that massive vectors have the momentum space propagator

$$\left[\frac{-i(g_{\mu\nu} - k_\mu k_\nu/\mu^2)}{k^2 - \mu^2 + i\epsilon} \right].$$

And $\langle 0 | A_\mu(x) | V(k, r) \rangle = \epsilon_\mu(k)^r e^{-ikx}$, so incoming vector mesons have $\epsilon_\mu^r(k)$ and outgoing have $\epsilon^{*r}(k)$.

We can couple the massive vector to other fields, e.g. to a fermion via $\mathcal{L}_{int} = -g\bar{\psi}\not{A}\Gamma\psi$, with $\Gamma = 1$ (vector) or $\Gamma = \gamma_5$ (axial vector). Gives Feynman rule that a vertex has a factor of $-ig\gamma^\mu\Gamma$.