10/17 Lecture outline

⋆ **Reading for today's lecture: Luke p. 65-80; Tong p. 35-41.**

• Last time: recall that $[\phi(x),\phi(y)] = D_1(x-y) - D_1(y-x)$, where

$$\langle 0|\phi(x)\phi(y)|0\rangle = D_1(x-y) \equiv \int \frac{d^3k}{(2\pi)^3 2\omega(k)} e^{-ik(x-y)}.$$

We considered the Green's function for $\partial^2 + m^2$ and introduced the Feynman propagator: go above the $k_0 = E_k$ pole and below the $k_0 = -E_k$ pole

$$D_F = \theta(x_0 - y_0) D_1(x-y) + \theta(y_0 - x_0) D_1(y-x)$$

$$D_F(x-y) \equiv \langle T\phi(x)\phi(y)\rangle = \begin{cases} \langle \phi(x)\phi(y)\rangle & \text{if } x_0 > y_0 \\ \langle \phi(y)\phi(x)\rangle & \text{if } y_0 > x_0 \end{cases}.$$

$$D_F(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} e^{-ik(x-y)},$$

• Define contraction of two fields $A(x)$ and $B(y)$ by $T(A(x)B(y)) - :A(x)B(y):$. This is a number, not an operator. Let e.g. $\phi(x) = \phi^+(x) + \phi^-(x)$, where ϕ^+ is the term with annihilation operators and ϕ^- is the one with creation operators (using Heisenberg and Pauli's reversed-looking notation). Then for $x^0 > y^0$ the contraction is $[A^+, B^-]$, and for $y^0 > x^0$ it is $[B^+, A^-]$. So can put between vacuum states to get that the contraction is $\langle TA(x)B(y)\rangle$. For example, in the KG theory the contraction of $\phi(x)$ and $\phi(y)$ is $D_F(x-y)$.

• Simple examples of interacting theory:

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi^2 - \mu^2\phi^2) - \rho(x)\phi$$

with $\rho(x)$ an external source forcing function. We'll show this theory is exactly solvable and gives probability for particle creation given by the Poisson distribution.

Next example:

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi^2 - \mu^2\phi^2) + (\partial\psi^\dagger\partial\psi - m^2\psi^\dagger\psi) - g\phi\psi\psi^\dagger.$$

Toy model for interacting nucleons and mesons. Treat last term as a perturbation.

• In QM we can use the S-picture, $i\hbar\frac{d}{dt}|\psi(t)\rangle = H|\psi\rangle$, or the H-picture, $i\hbar\frac{d}{dt}\mathcal{O}(t) = [\mathcal{O}, H]$. In interacting theories, it is useful to use the hybrid, interaction picture. Write

$H = H_0 + H_{int}$. We use H_0 to time evolve the operators, and H_{int} to time evolve the states:

$$i\frac{d}{dt}\mathcal{O}(t) = [\mathcal{O}, H_0], \qquad i\frac{d}{dt}|\psi(t)\rangle = H_{int}|\psi(t)\rangle.$$

For example, we'll take H_0 to be the free Hamilton of KG fields, with only the mass terms included in the potential. Again, this is free because the EOM are linear, and we can solve for $\phi(x)$ by superposition. As before, upon quantization, the fields become superpositions of creation and annihilation operators. The states are all the various multiparticle states, coming from acting with the creation operators on the vacuum.

• Compute probabilities from squaring amplitudes, and amplitudes from $\langle f(t = +\infty)|i(t=-\infty)\rangle = \langle f|S|i\rangle = \langle f|U(\infty, -\infty)|i\rangle$. Naively, $U(t_f, t_i) = \exp(-\frac{i}{\hbar}\int_{t_i}^{t_f} H_{int}(t)dt)$, but have to be careful about H_{int} not commuting at different times. Get time ordering.

• Dyson's formula. Compute scattering S-matrices. Consider asymptotic in and out states, with the interaction turned off. Time evolve, with the interaction smoothly turned on and off in the middle.

$$|\psi(t)\rangle = Te^{-i\int d^4x \mathcal{H}_I}|i\rangle.$$

Derive it by solving $i\frac{d}{dt}|\psi(t)\rangle = H_I(t)|\psi(t)\rangle$ iteratively:

$$|\psi(t)\rangle = |i\rangle + (-i)\int_{-\infty}^{t} dt_1 H_I(t_1)|\psi(t_1)\rangle$$

$$|\psi(t_1)\rangle = |i\rangle + (-i)\int_{-\infty}^{t_1} dt_2 H_I(t_2)|\psi(t_2)\rangle$$

etc where $t_1 > t_2$, and then symmetrize in t_1 and t_2 etc., which is what the T time ordering does.