10/29 Lecture outline

⋆ **Reading for today's lecture: Luke p. 65-80; Tong p. 35-41.**

• Last time: some amplitudes in our nucleon + meson toy model, via

$$\langle f|(S-1)|i\rangle = \langle f|Te^{-i\int d^4x\mathcal{H}_I(x)}|i\rangle \equiv i\mathcal{A}_{fi}(2\pi)^4\delta^{(4)}(p_f - p_i).$$

Examples: meson decay $\phi \to N + \bar{N}$ has $\mathcal{A}(\phi \to N + \bar{N}) = -g + O(g^3)$,
$N + N \to N + N$, to $\mathcal{O}(g^2)$:

$$\mathcal{A} = (-ig)^2\left[\frac{1}{(p_1 - p_1')^2 - \mu^2} + \frac{1}{(p_1 - p_2')^2 - \mu^2}\right].$$

Explicitly, in the CM frame, $p_1 = (\sqrt{p^2+m^2}, p\widehat{e})$ and $p_2 = (\sqrt{p^2+m^2}, -p\widehat{e})$, $p_1' = (\sqrt{p^2+m^2}, p\widehat{e}')$, $p_2' = (\sqrt{p^2+m^2}, -p\widehat{e}')$, where $\widehat{e}\cdot\widehat{e}' = \cos\theta$, and get

$$\mathcal{A}(N+N \to N+N) = g^2\left(\frac{1}{2p^2(1-\cos\theta)+\mu^2} + \frac{1}{2p^2(1+\cos\theta)+\mu^2}\right) + O(g^4)$$

As we'll discuss, scattering by ϕ exchange leads to an attractive Yukawa potential.

• Recall how we got the above answer in the previous lecture. We expand $\exp(-ig\int d^4x\mathcal{H})$ and compute the time ordered expectation values using Wick's theorems, with the contractions giving factors of $D_F(x_1 - x_2)$. Doing this, we get a $\int d^4x$ for each factor of $-ig$ and a d^4k for each internal contraction. Draw a picture in position space. Let E be the number of external lines, i.e. the number of incoming + outgoing particles. (We saw last time that $[\mathcal{A}] = 4 - E$.) It is easier to think about everything in momentum space. Then the $\int d^4x$ for each vertex gives a $(2\pi)^4\delta^4(p_{total,\ in})$.

• Feynman rules! Each vertex gets $(-ig)(2\pi)^4\delta^4(p_{total\ in})$, each internal line gets $\int\frac{d^4k}{(2\pi)^4}D_F(k^2)$, where D_F is the propagator, e.g. $D_F(k^2) = \frac{i}{k^2-m^2+i\epsilon}$. Result is $\langle f|(S-1)|i\rangle$, so divide by $(2\pi)^4\delta^4(p_F - p_I)$ to get $i\mathcal{A}_{fi}$.

If the diagram has no loops, the momentum conserving delta functions fix all internal momenta in terms of the external ones. When the diagram has $L \neq 0$ loops, the procedure above yields integrals over the internal momenta of the loops. (Note that if a diagram has I internal lines and V vertices, then there are I momentum integrals, and V momentum conserving delta functions; one of these becomes overall momentum conservation, so there are $L = I - V - 1$ momentum integrals left to do, and L is the number of loops in the diagram.) Any loop momentum integrals require renormalization, which we'll discuss later (next quarter), so for now we'll just consider "tree-level" contributions, associated with diagrams without loops, $L = 0$.

• More examples:

(1) $N(p_1) + \bar{N}(p_2) \to N(p_1') + \bar{N}(p_2')$ has

$$i\mathcal{A} = (-ig)^2 \left(\frac{i}{(p_1 - p_1') - \mu^2} + \frac{i}{(p_1 + p_2) - \mu^2} \right).$$

(2) $N(p_1) + \bar{N}(p_2) \to \phi(p_1')\phi(p_2')$ has

$$i\mathcal{A} = (-ig)^2 \left(\frac{i}{(p_1 - p_1') - m^2} + \frac{i}{(p_1 - p_2') - m^2} \right).$$

(3) $N(p_1) + \phi(p_2) \to N(p_1') + \phi(p_2')$ has

$$i\mathcal{A} = (-ig)^2 \left(\frac{i}{(p_1 - p_2') - m^2} + \frac{i}{(p_1 + p_2) - m^2} \right).$$

• Mandelstam variables. $s = (p_1+p_2)^2$, $t = (p_1-p_1')^2$, $u = (p_1-p_2')^2$, with $s+t+u = 4m^2$ (more generally, $s+t+u = \sum_{i=1}^4 m_i^2$). In CM, $s = 4E^2$, $t = -2p^2(1-\cos\theta)$, and $u = -2p^2(1+\cos\theta)$.

• Crossing symmetry, CPT. Write $1 + 2 \to \bar{3} + \bar{4}$. Take all momenta incoming, $\mathcal{A}(p_1, p_2, p_3, p_4)$, with $p_1 + p_2 + p_3 + p_4 = 0$ and use $s = (p_1 + p_2)^2$, $t = (p_1 + p_3)^2$ and $u = (p_1 + p_4)^2$. Note $s + t + u = \sum_{n=1}^4 m_n^2$. The process $1 + 2 \to \bar{3} + \bar{4}$ is kinematically allowed for $s > 4m^2$, $t < 0$, $u < 0$. If instead $u > 4m^2$, it's the process $1 + 3 \to \bar{2} + \bar{4}$.