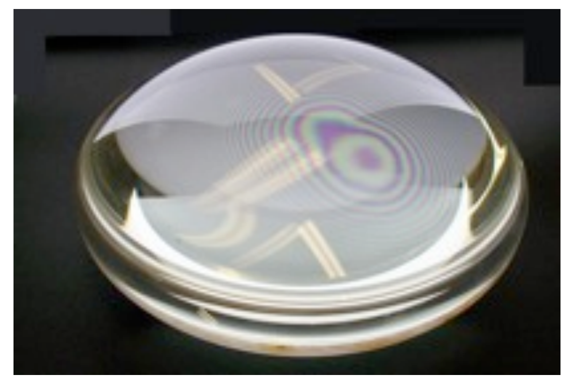
Light propagation

Ken Intriligator's week 7 lectures, Nov. 12, 2013

What is light?

Old question: is it a wave or a particle?
Quantum mechanics: it is both!

1600-1900: it is a wave.
~1905: photons

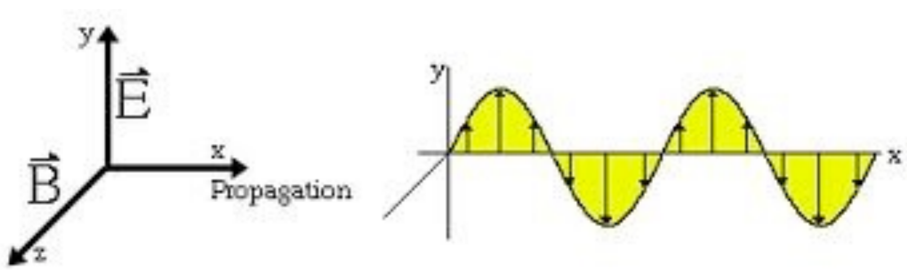


Wave: wavelength determines color, amplitude determines intensity, brightness.

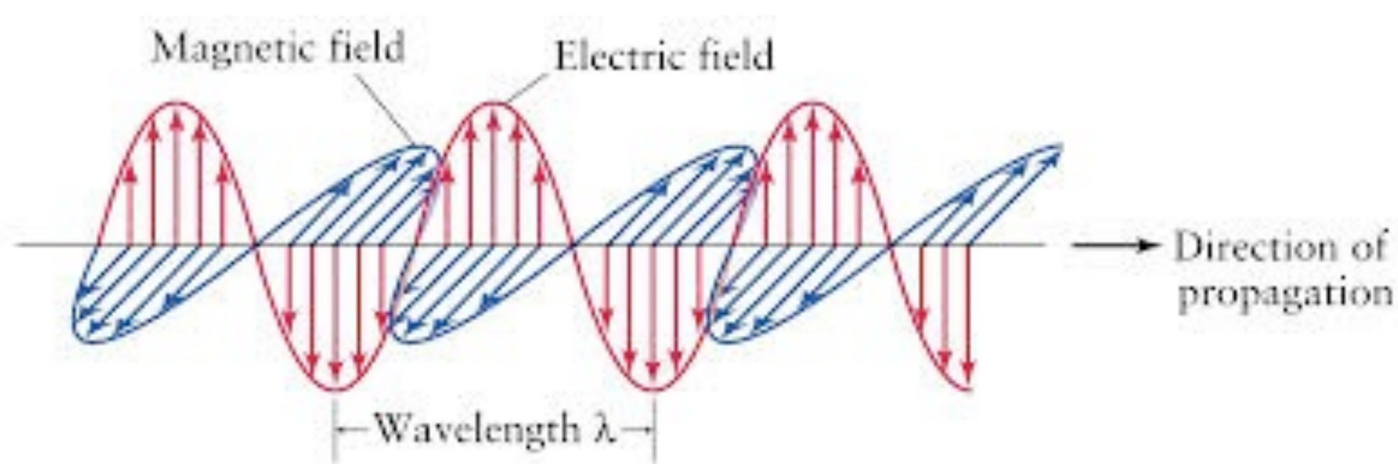
Photons: massless particles, energy determined by the wavelength, color. Intensity determined by number of photons. Won't discuss photons further in 2C.

Physics 2B stuff:

- Changing B field acts as a source to make EMF (that's how electricity is generated).
- Changing E field also acts as a source for B.
- Light is an E & B wave, each flip-flopping to make the other. Wave moves at v=c, the speed of light.



$$\vec{k} \propto \vec{S} \propto \vec{E} \times \vec{B}$$



(The details)

Maxwell eqns: $$\nabla \cdot \vec{E} = \cancelto{0}{\rho}/\epsilon_0, \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Empty space, no sources $$\nabla \cdot \vec{B} = 0, \qquad \nabla \times \vec{B} = \cancelto{0}{\mu_0 \vec{J}} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}.$$

E.g. take $\vec{E} = \vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega t)$ & $\vec{B} = \vec{B}_0 \cos(\vec{k} \cdot \vec{r} - \omega t)$

$$\nabla \cdot \vec{E} = -(\vec{k} \cdot \vec{E}_0) \sin(\vec{k} \cdot r - \omega t) \qquad \nabla \times \vec{E} = -(\vec{k} \times \vec{E}_0) \sin(\vec{k} \cdot r - \omega t)$$

Need: $\vec{k} \cdot \vec{E}_0 = \vec{k} \cdot \vec{B}_0 = 0$ E and B are transverse to the wave velocity.

$$\vec{B}_0 = \omega^{-1} \vec{k} \times \vec{E}_0$$

$$\omega = c|\vec{k}| \qquad c = 1/\sqrt{\mu_0 \epsilon_0} \approx 3 \times 10^8 m/s$$

light speed

$$c \equiv 299,792,458 m/s \quad \approx 3 \times 10^8 m/s$$

Modern definition of meter. Instead of using a special ruler to define the meter, use light! They could have defined it to be a simpler number for c, but they chose it close to an earlier, historic definition of the meter.

Example of light wave

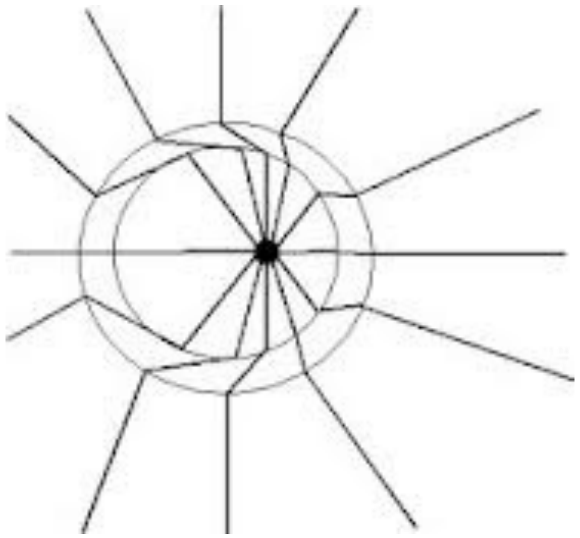
$$\vec{E} = E_0 \widehat{y} \cos(kx - \omega t)$$

$$\vec{B} = \frac{E_0}{c} \widehat{z} \cos(kx - \omega t)$$

"plane wave" moving in the + x direction.

Maxwell's eqns. require $v_\phi \equiv \omega / k = c$

Light wave carries energy and momentum (recall the Poynting vector). It can also carry angular momentum. Light waves are created by accelerating charges.



Light as E & M waves

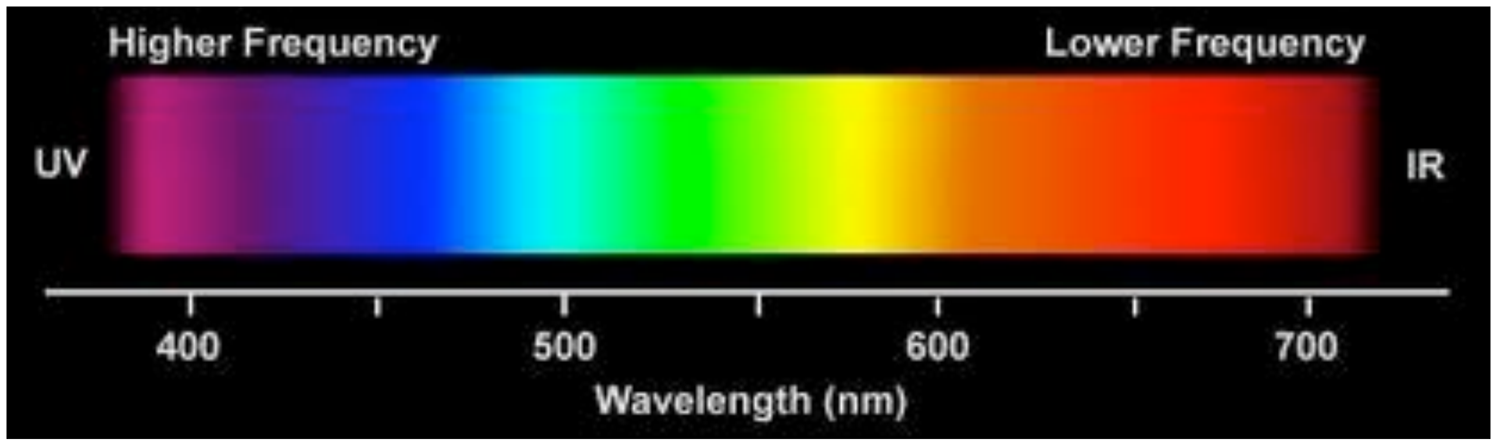
E.g. $\vec{E} = E_0 \widehat{y} \cos(kx - \omega t)$

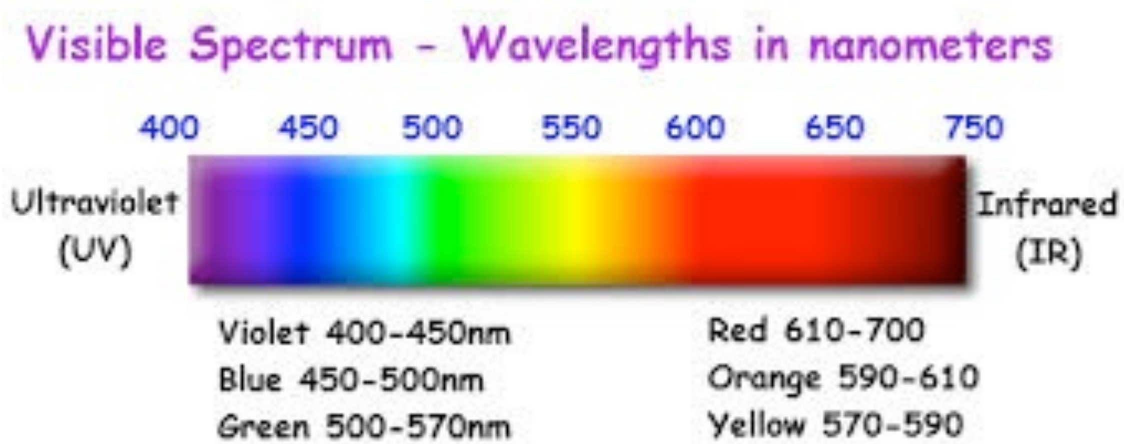
Frequency (equivalently wavelength) determines the **color** of the light.

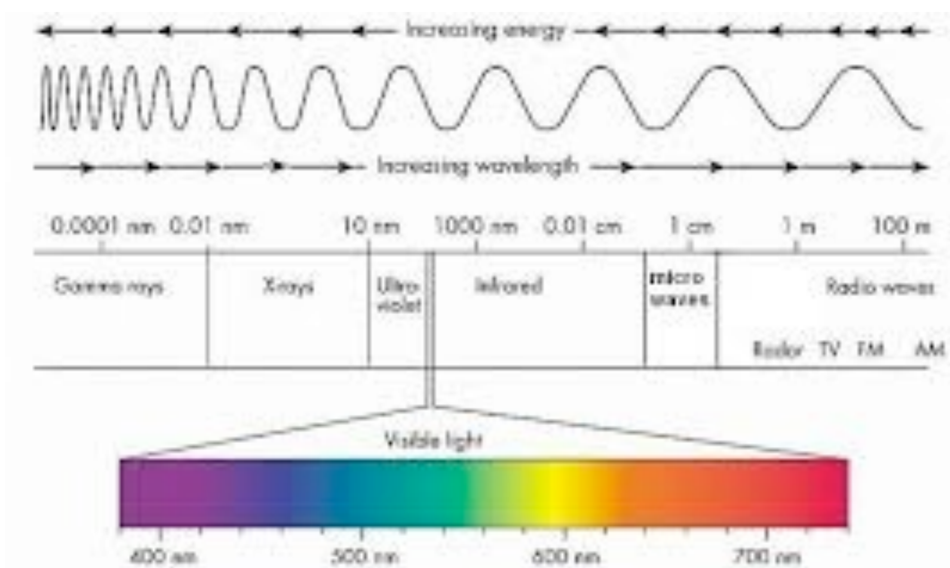
Polarization

Amplitude of wave. Intensity (**brightness**) of the light, i.e. how much power it carries, is prop. to the square of the amplitude (similar to sound waves etc.).

Light spectrum



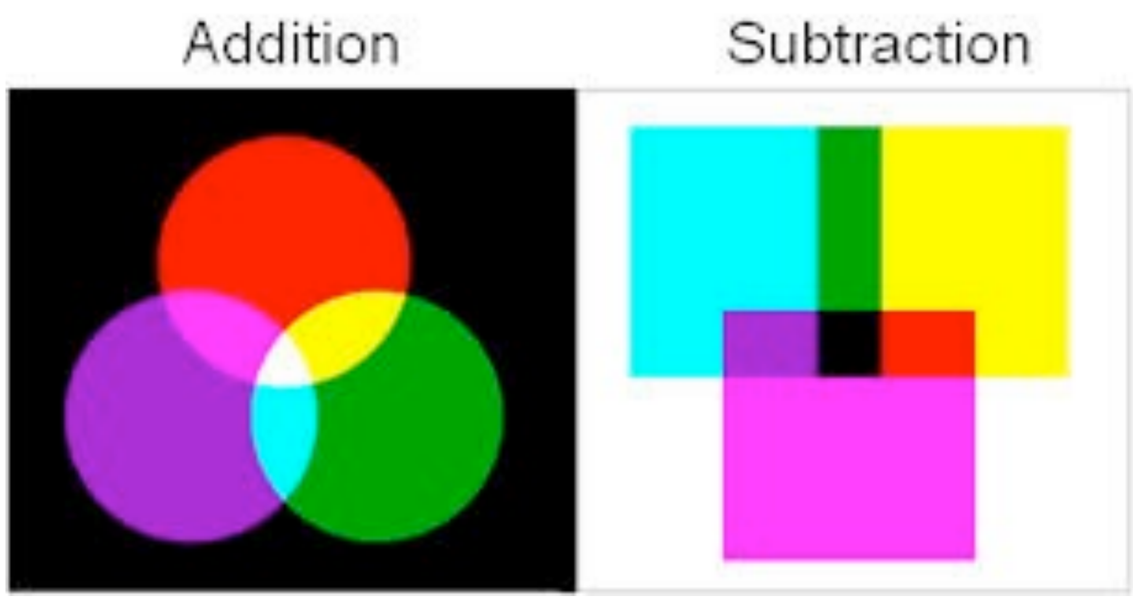


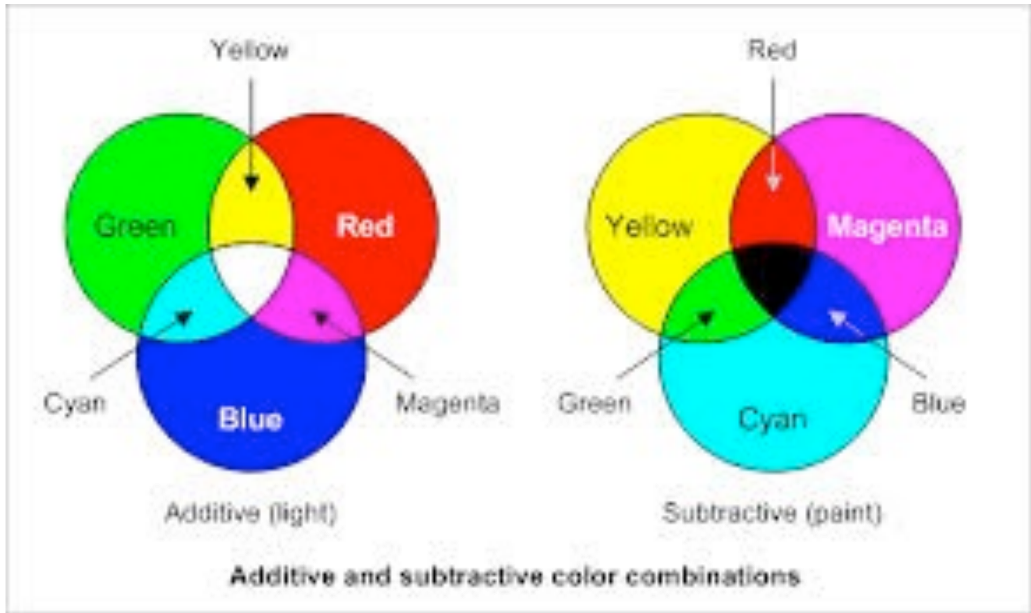


“What’s your favorite wavelength?”

Aside: paint’s color comes from absorbing more light on opposite side of spectrum.

Aside: color addition





Aside: bee's eyes!

Bees see UV spectrum (small wavelength) that we cannot see. Flowers evolve to look especially pretty for the bees, with bullseye targets, visible in the UV spectrum.

humans: 390-750nm

bees: 300-650nm

human's view

Bee's view

human's view

Bee's view

Light in clear materials

Maxwell's eqns. in materials: $\mu_0 \to \mu$ Magnetic permeability

$\epsilon_0 \to \epsilon$ and electric polarizability.

If you've seen this in Phys 2B, great - if not, don't worry about it. We don't really need much here.

All we need to know is that we again find Maxwell's equations admits wave solutions, but the velocity of the wave is modified by the material:

$$v = \frac{1}{\sqrt{\mu\epsilon}} \equiv c/n.$$

Index of refraction n

Light effectively moves slower than c in materials, because it has to bounce around through an obstacle course of atoms. Amount of slowing depends on the material, it's density etc. Write it as

$$v = c/n \le c \qquad n \ge 1$$

(There can be exceptions!)

Aside:

Actually, it's possible to design exotic "metamaterials," with negative index of refraction! Pioneered here at UCSD in 2000 by Shelly Schultz and David Smith (my friend, and former college roommate here at UCSD).

Another aside

Modern methods to make n so big that light moves at a snail's pace, or even stopped all together! L. Hau:

http://www.youtube.com/watch?v=-8Nj2uTZc10

Light wave with index n

Traveling wave with frequency $\omega = 2\pi/T$

and wavenumber $k = |\vec{k}| = 2\pi/\lambda$

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \frac{c}{n}$$

with

$$\omega = \omega_{vac}$$

$$k = nk_{vac}$$

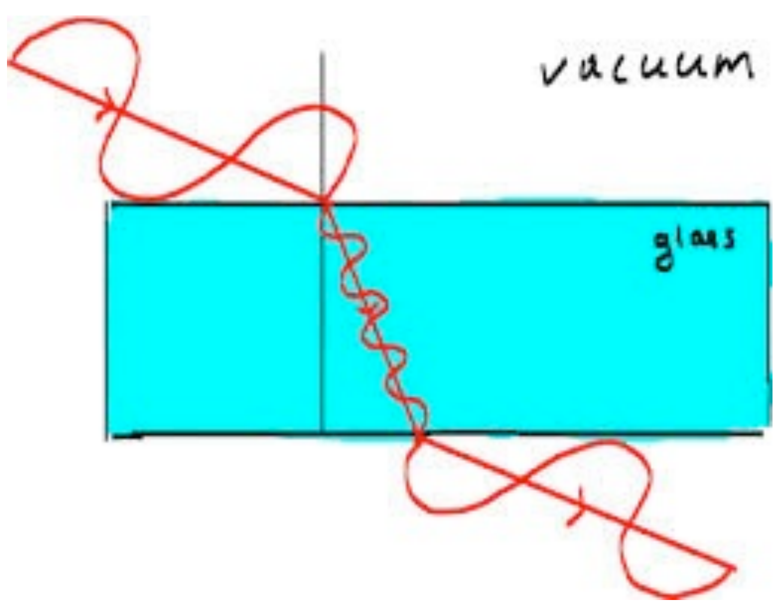
$$\lambda = \lambda_{vac}/n$$

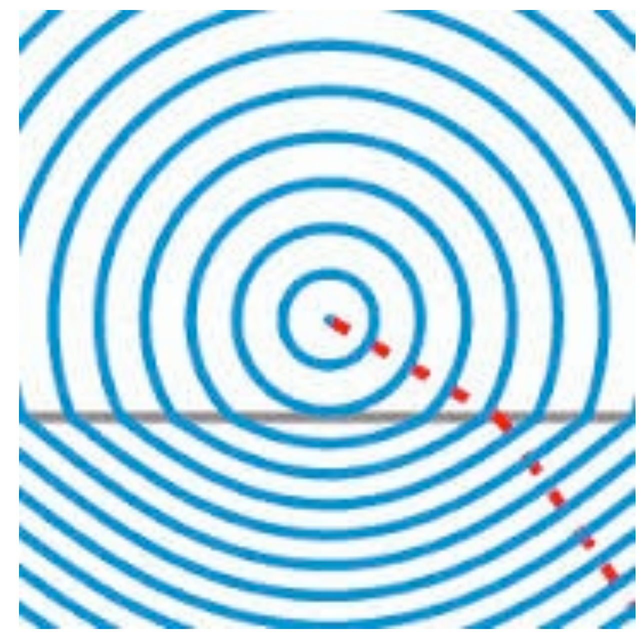
vacuum | index n

Same time dependence on two sides,
only the spatial part is affected.

wavelength, index n

$$\lambda = \lambda_{vac}/n$$

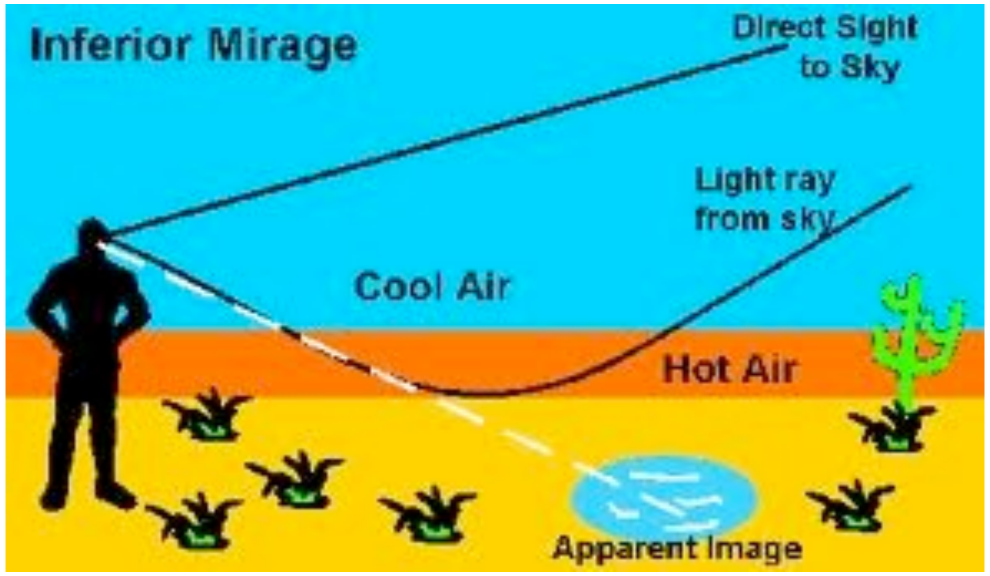


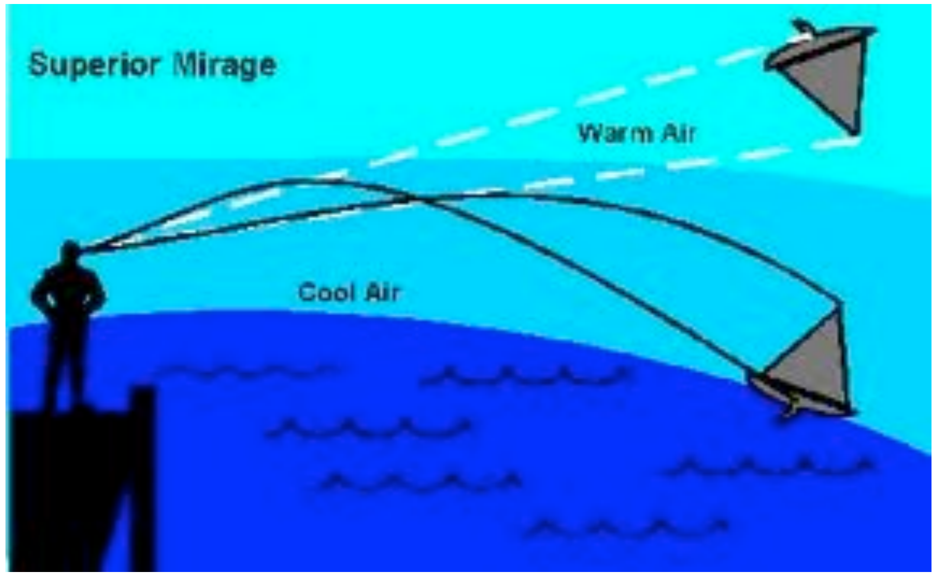


leads to
Snell's law

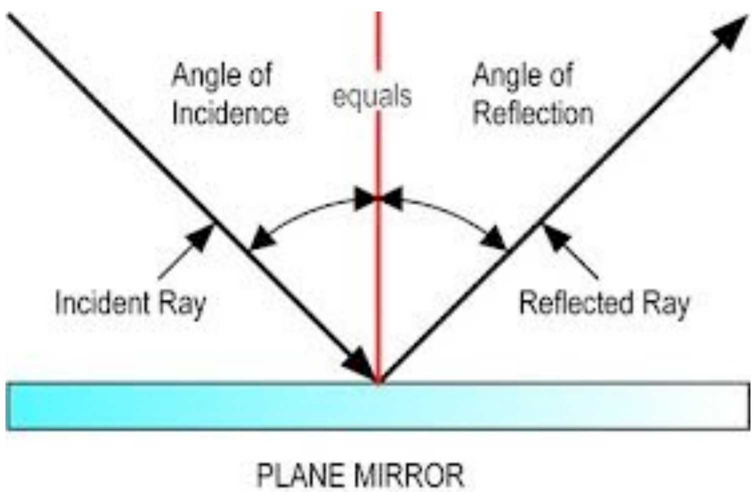
Fermat's principle

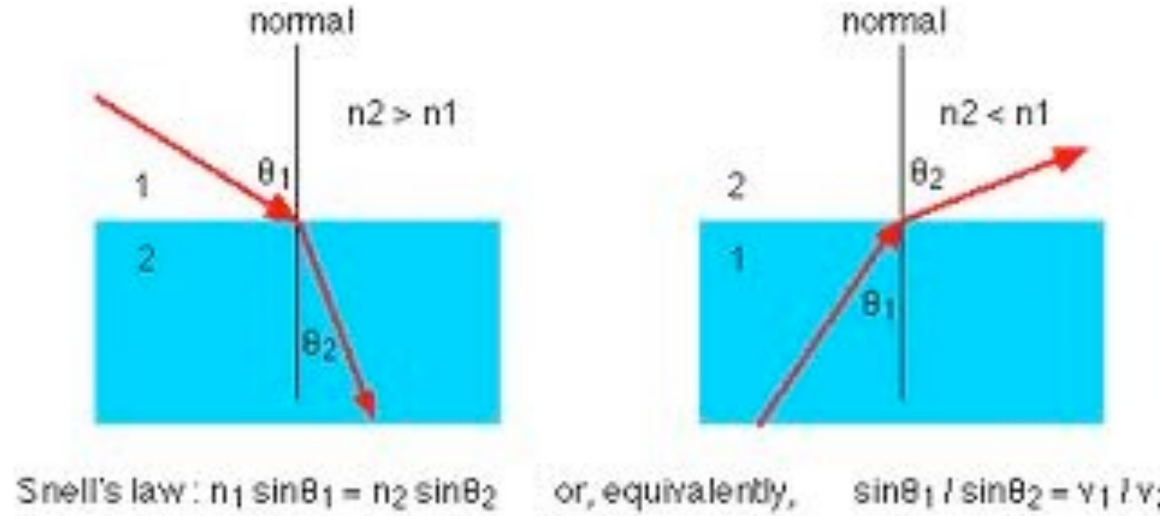
"Light always takes the path of **shortest time**."
If n is unchanged everywhere, the shortest time path is the same as the shortest distance path. Even if light goes faster in some regions than others (n is larger in the slower regions), light always manages to find and take the quickest path. Example: mirages





Reflection & refraction

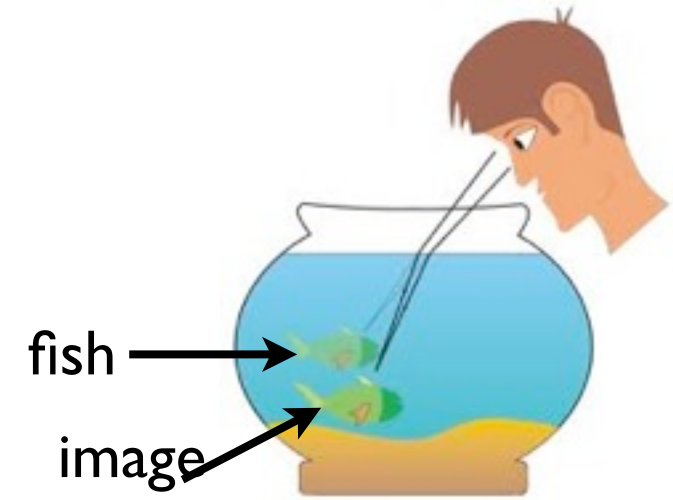




Fermat: the path always (locally) minimizes the **time** needed for light to go from source to observer. Shortens distance on slower side, to reduce time.

$$\longrightarrow \qquad n_1 \sin\theta_1 = n_2 \sin\theta_2$$ "Snell's law"

Refraction, cont.



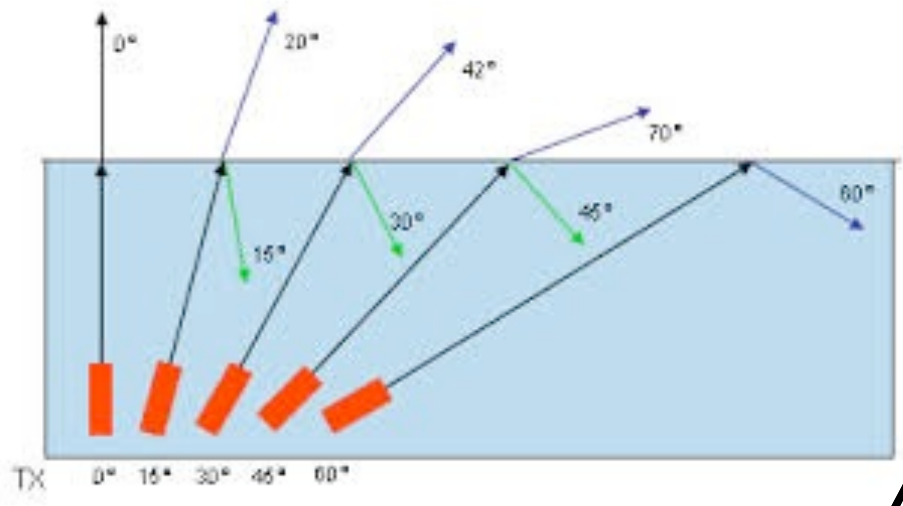
"Where is that fish?"

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_1 = \frac{n_2}{n_1}\sin\theta_2 \overset{!}{\leq} 1$$

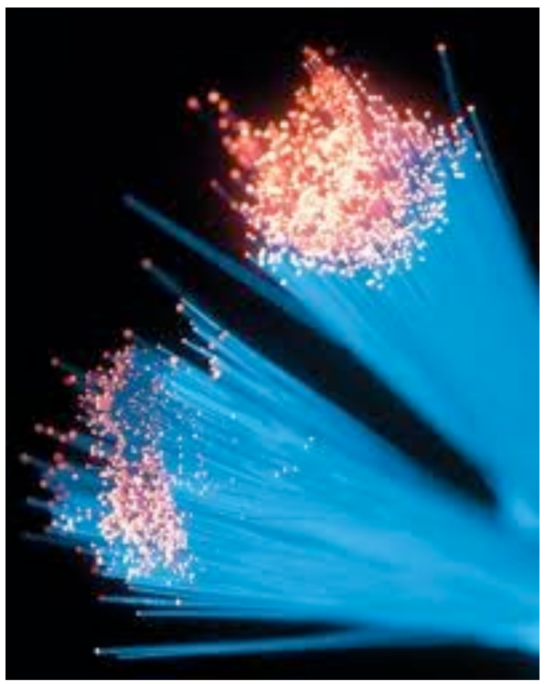
No refraction if this is >1!

$$\boxed{\sin\theta_2^{crit} = n_1/n_2}$$

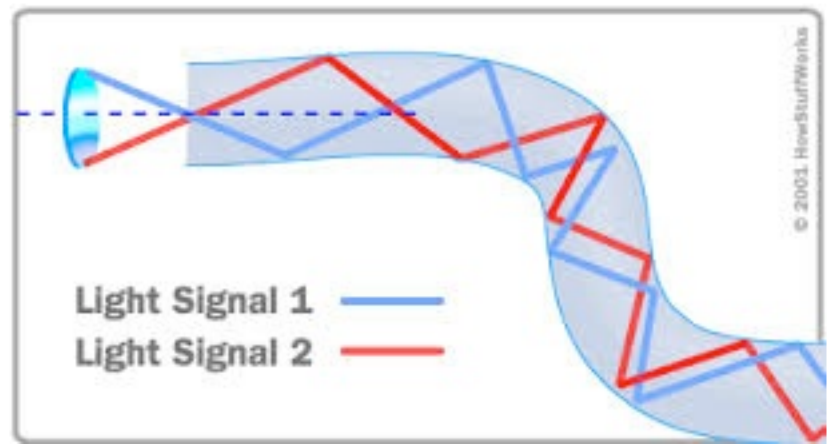


Total internal reflection inside slower region if the angle is big enough. Try it out in a pool. From underwater, look up, out of the water: see a mirrored surface at big angles.

Fiber optics



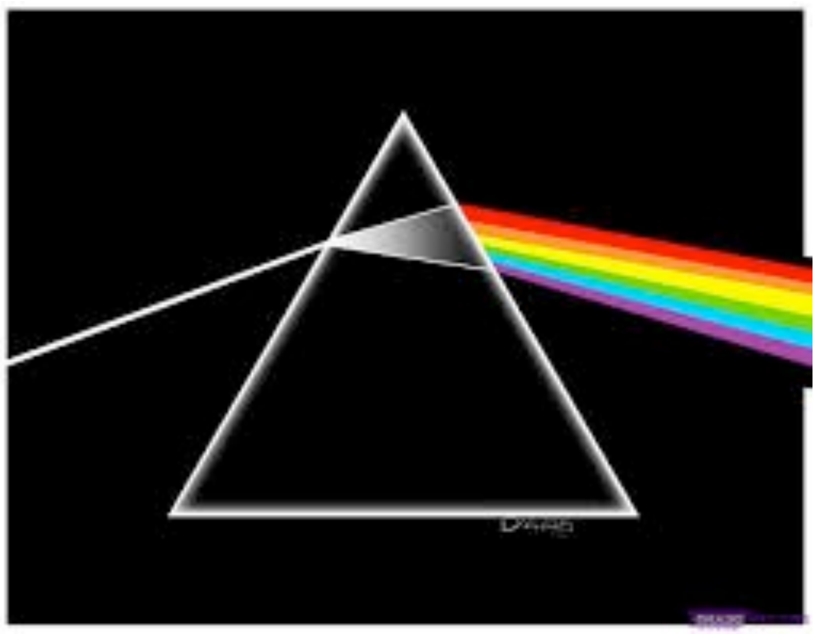
Total internal reflection.

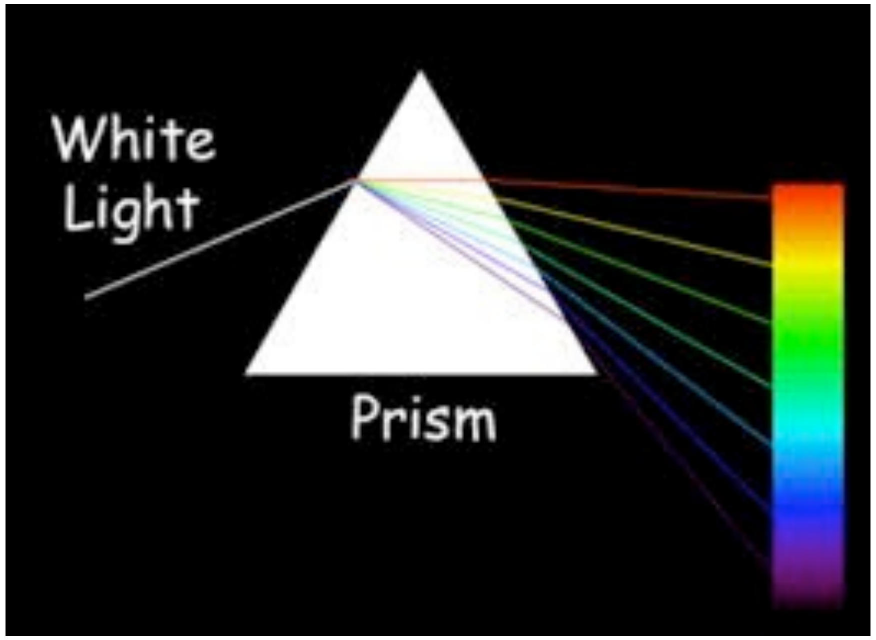


Basis for global, landline communication / data transfer.

Dispersion

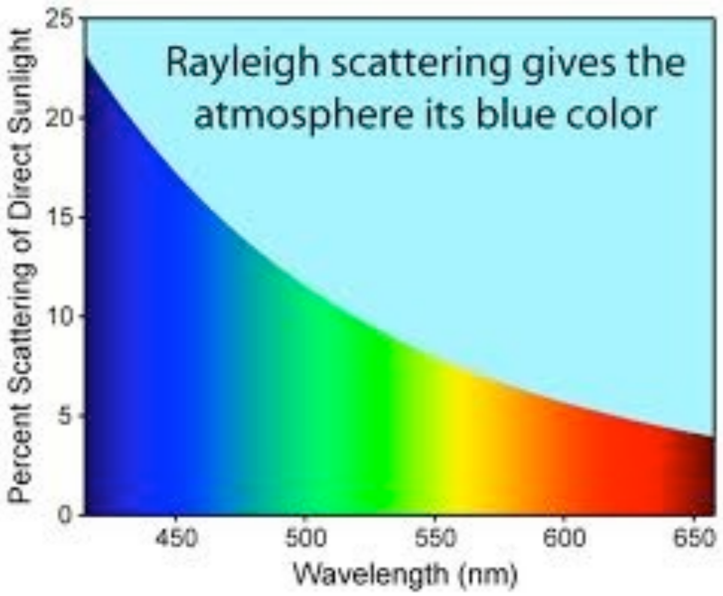
If $n = n(\omega)$ or, equivalently, $n = n(\lambda)$





Shorter wavelengths have bigger index of refraction (can picture that they collide into more atoms, so they are slowed more than longer wavelenghts).
So shorter wavelengths are bent more by the prism.

Colors in the sky



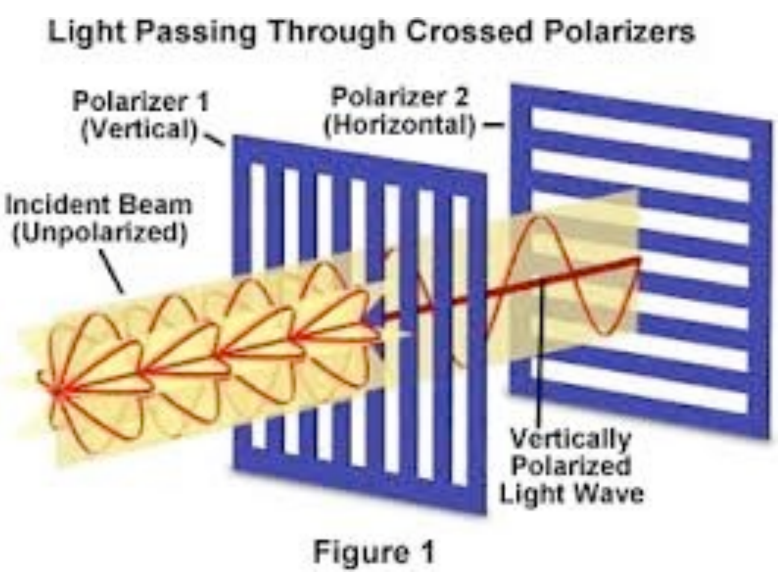
Blue light is scattered more by air molecules than red light. Makes sky look blue. At sunset, sunlight goes through more air, more blue is scattered out, so the light passing through has the opposite color: red.

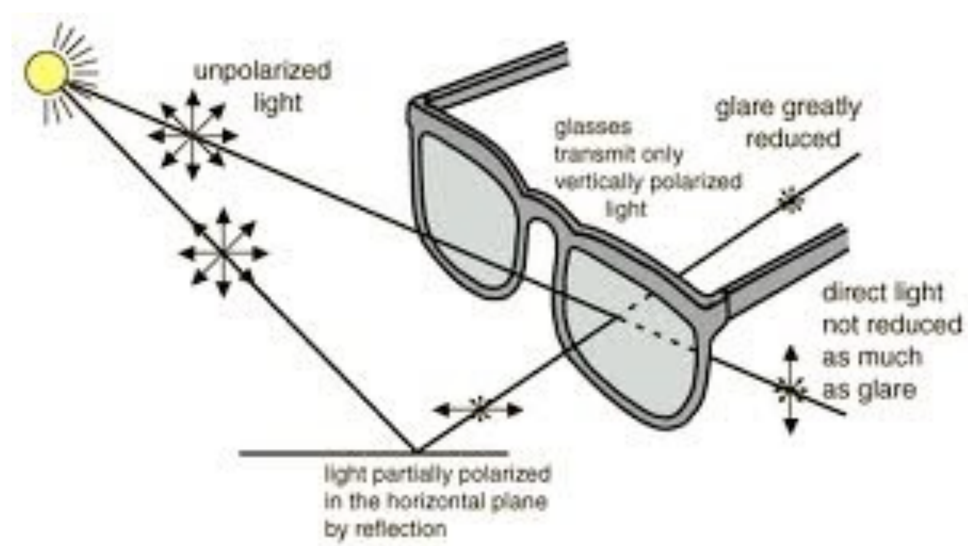
Raleigh scattering: $\sigma \sim \lambda^{-4}$

Polarization

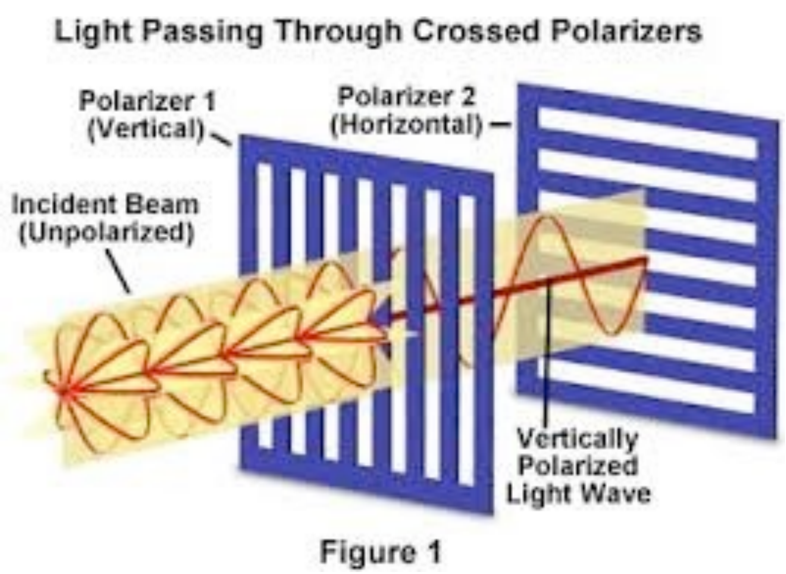
E.g.: $\vec{E} = E_0 \widehat{y} \cos(kx - \omega t)$ Polarized light wave.

Most light sources (light bulbs, sun, etc) make unpolarized light, which is a superposition of randomly polarized light waves. Polarizing filters block the components except those with E along (actually perpendicular to) the polarizer's axis.





Polarization, cont.



$$E_2 = E_1 \cos\theta$$

Angle between initial polarization of E, and the direction of polarizer slits.

Amplitude afterwards.

Amplitude before

$$I_2 = I_1 \cos^2\theta$$

Initially unpolarized light: $I_2 = \frac{1}{2} I_1$

Perpendicular polarizers: $I_{out} = 0$

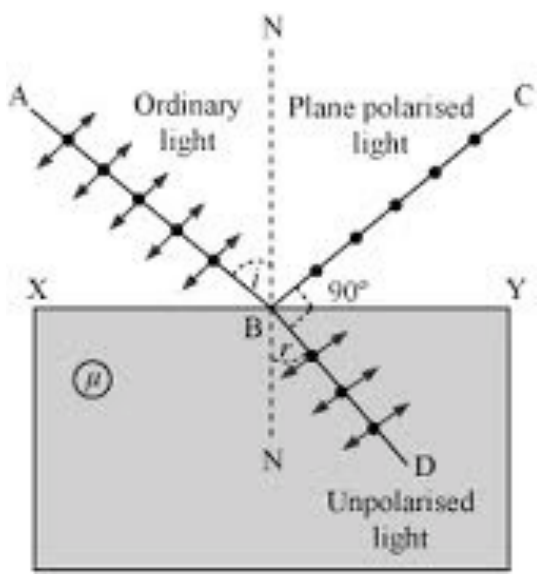
Perpendicular polarizers, with another non-perpendicular one in between: $I_{out} \neq 0$

Brewster's law

Reflected waves want to have their E parallel to the reflecting surface. Like skipping stones on water. Drawing picture of plane of incidence, E is better-reflected if it's perpendicular to that plane. If the two angles in Snell's law add to make a right-angle(90 degrees), the reflected light is 100% polarized (perp. to incidence plane).

$$n_1 \sin\theta_B = n_2 \sin(\frac{\pi}{2} - \theta_B) = n_2 \cos\theta_B$$

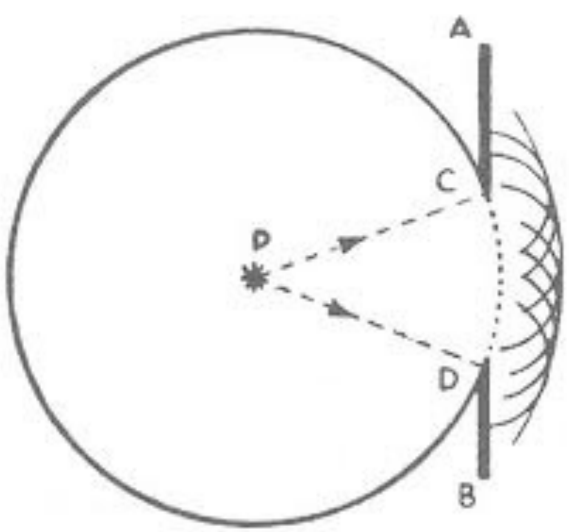
$$\tan\theta_B = n_2/n_1$$

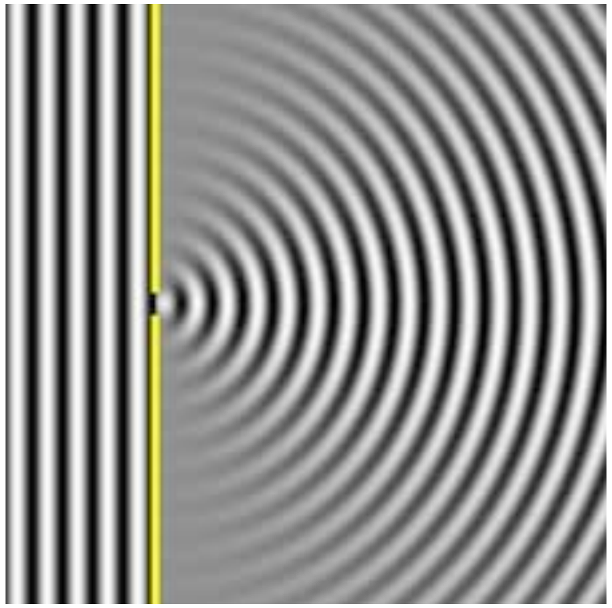


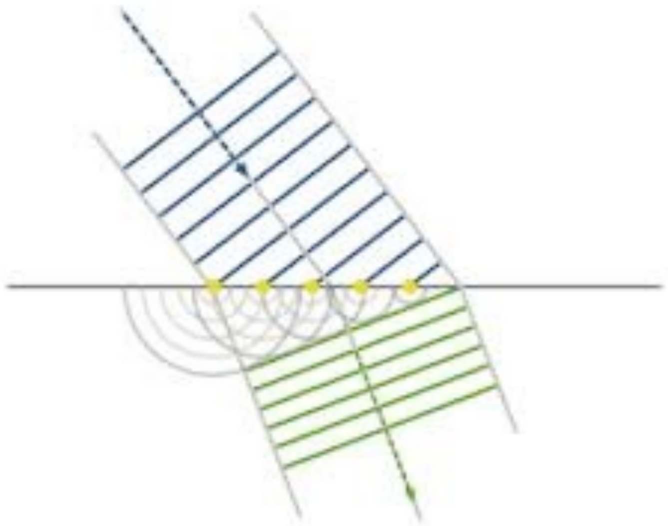
Huygen's principle

Every point on wave front acts as a point source for the wave at later times. Later wave front all these sources.

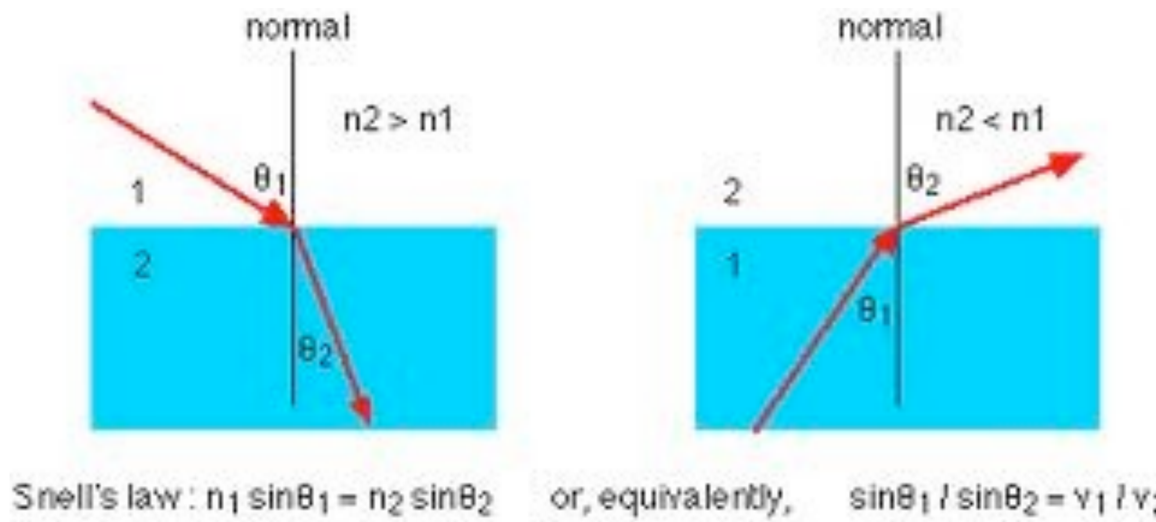
Can show that this leads to Snell's law, and the law of reflection, etc.

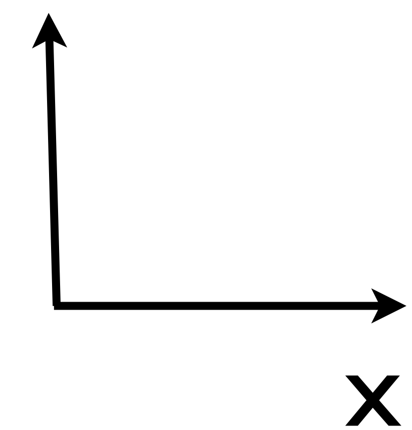






Yet another way to get Snell's law:





$$k_{x,1} = k_{x,2}$$

related to conservation of x-compt of photon's momentum (phy2d) $\vec{p} \propto \vec{k}$.

$$k_{i,x} = k_i \sin\theta_i$$

$$k_i = k_{vac} n_i$$