- What is a fluid

- ρ & p

- F atm on palm of hand p scalar

- bike vs car p. Ear $k\Delta x = \Delta p A$

$\Delta p = 2.8 \times 10$
to 28

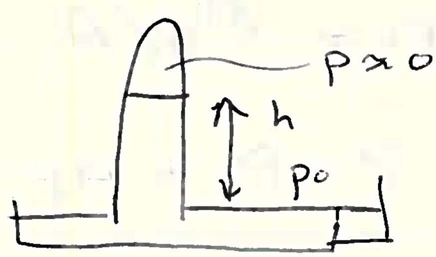
- $d\vec{F} = -p\,\hat{n}\,dA = -\nabla p\, dV$

— $p = p_0 + \rho g h$ (ρ const)

— 100 m vs 5 atm

$$\Delta h = 1.01 \times 10^5 \text{ Pa} / (10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)$$

$$= 10.3 \text{ meters for each atm}$$

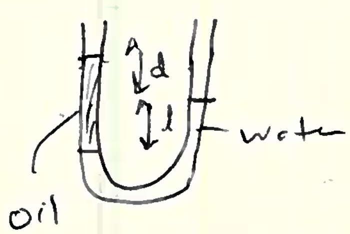
— 

$p_0 = \rho g h$

$\rho_{H_2O} = 10^3 \text{ kg/m}^3 \rightarrow$

$h \approx 10$ meters

Hg $\rho = 1.4 \times 10^4 \text{ kg/m}^3$ $h \approx .7$ meters

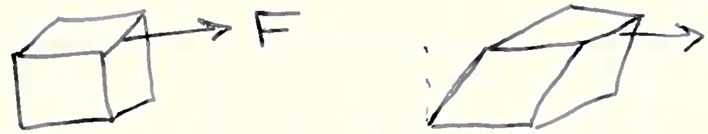
— 

$$p = p_0 + \rho_x g (l + d)$$
$$= p_0 + \rho_w g l$$

$$\rho_x (l + d) = \rho_w l$$

Fluids

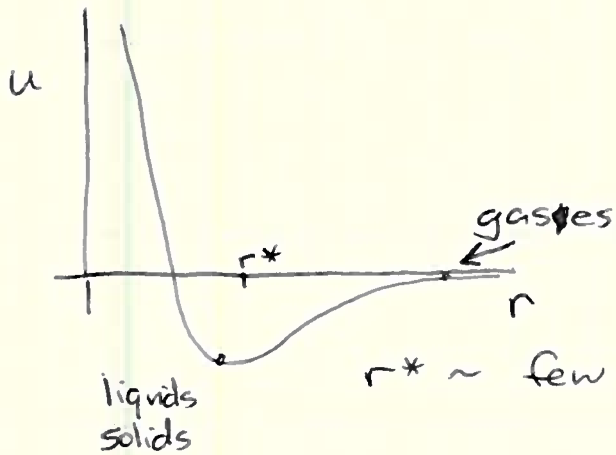
(vs solids) they flow in response to shear stresses



no restoring force.

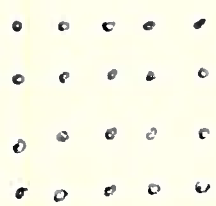
Do have restoring force for forces $\perp$ to face.

On micro level, inter-molecule potential



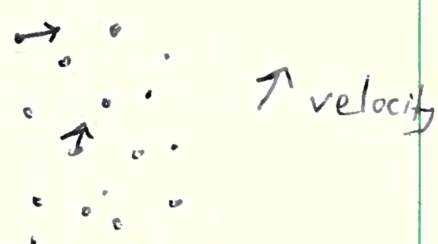
$r^* \sim$ few Å (10^{-10} m)

Solids



lattice - order

Fluids



disorder

shear restoring force → preserves order

no memory of position

Density

$$\rho = \frac{dm}{dV} \xrightarrow{\text{uniform}} m/V \qquad [\rho] = \text{kg}/\text{m}^3$$

e.g. best lab vac		10^{-17}	kg/m3	
air	20°C + 1atm	1.21		→ compressible
	20°C + 50atm	60.5		
water	20°C + 1atm	$.998 \times 10^3$		→ incompressible
	20°C + 50atm	1.000×10^3		

$\boxed{\text{black hole} \sim 10^{19}}$

Pressure

$$p = \frac{dF}{dA} \xrightarrow{\text{Uniform}} F/A$$

Units $[p]$ = Newton/meter^2 $(\sim M/LT^2)$

1 Newton/meter^2 $\equiv$ 1 Pa "pascal"

e.g. $p_{atmosphere}$ (at sea level) 1×10^5 Pa

$\equiv$ 1 atm = 14.7 lb/in^2

- Total force of atmosphere on palm of your hand:

  $A \approx 10\,\text{in}^2$ $\quad F = (14.7)(10) \approx 147$ lbs

- Auto tire pressure (in excess of atmosphere pressure)

Area A of tire on ground $\sim (15\,\text{cm})^2 \times 4$

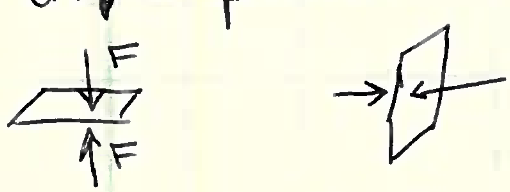
$\sim .1\,\text{m}^2 \sim 160\,\text{in}^2$

F = weight of car $\sim 2\times10^3\,\text{kg} \times (9.8) \sim 2\times10^4\,\text{N} \sim 4000\,\text{lbs}$

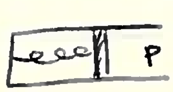
$\therefore$ pressure $p = 2\times10^5$ Pa ~ 25 lbs/in^2

Hydrostatics: Fluids at rest. No shear forces, all forces $\perp$ to every surface. Force element, dF_p, on any surface element dA is

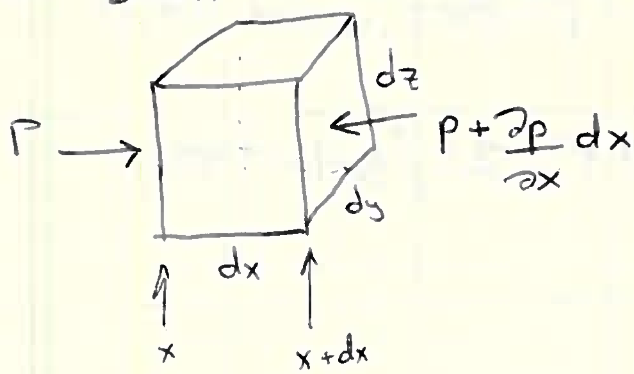
$$dF_p = -p\,\hat{n}\,dA \qquad (\hat{n}: \text{outward normal vector})$$



(magnitude of F indep. of orientation) $\quad$ $\underline{p = \text{scalar}}$

Pressure sensor:  $\quad F = k\Delta x = pA \qquad p = k\Delta x/A$

Force on small volume due to pressure



etc $\longrightarrow d\vec{F}_p = -\vec{\nabla} p \, d$

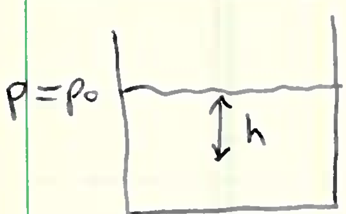
force pushes in dir. of decreasing pressure → equilibrium

In equilibrium, with no outside forces, $p = \text{const}$

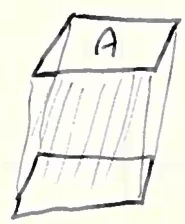
With gravity, $\vec{F}_{total} = \vec{F}_p + \vec{F}_{grav} = 0$

$$\vec{\nabla} p = \rho \vec{g} \implies p(z) = p(z_0) - \rho g (z - z_0)$$

(if ρ & $\vec{g}$ const.)
"incompressible"



$$p = p_0 + \rho g h$$



weight p_0

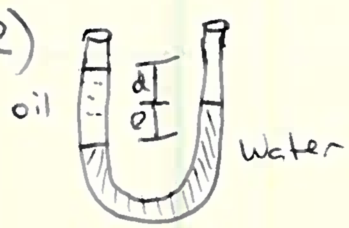
$p_0 A + \rho h A g$

Eg: 1) some waterproof watches quote max depth in Atm
What is that in meters?

$$1\,\text{atm} = \Delta p = \rho_{H_2O} \, g \, \Delta h \longrightarrow \Delta h = \Delta p / \rho_{H_2O} g$$

$$\Delta h = 1.01 \times 10^5 \, \text{Pa} / (10^3 \, \text{kg/m}^3)(9.8 \, \text{m/s}^2) = 10.3 \, \text{m} \sim 34$$

#2)



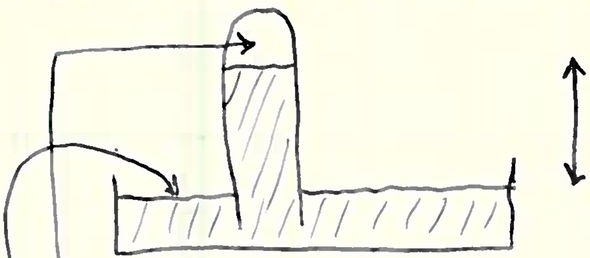
$$g \, \rho_{oil} (d + l) = g \, \rho_{H_2O} \, l$$

$$\implies \frac{d}{l} = \frac{\rho_{H_2O} - \rho_{oil}}{\rho_{oil}} \quad \text{indep. of } g$$

Measuring pressure with a Barometer:

i) fill long glass tube with liquid

ii) invert in a dish of the liquid



height h above level in dish

pressure here is $p \approx 0$ (almost vacuum)

pressure p here is p_0 = atmosphere pressure

equilibrium: $p_0 = \rho g h$

height h of barometer: $h = p_0 / \rho g$

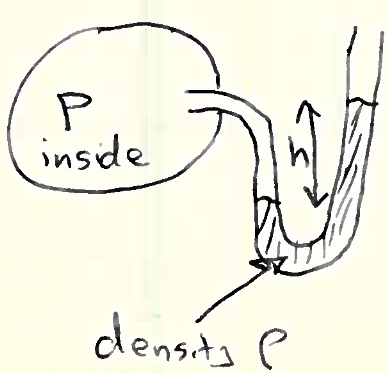
If liquid is water, using $p_0 = 1 \times 10^5$ Pa
and $\rho_{H_2O} = 10^3$ kg/m^3 $h \approx 10$ meters – too big!

Better to use mercury Hg, $\rho_{Hg} = 1.4 \times 10^4$ kg/m^3

so now $h \approx .7$ meters – more manageable.

<u>Open tube manometer</u>: measure pressure <u>difference</u> between fluid and atmosphere

p_0 at open end

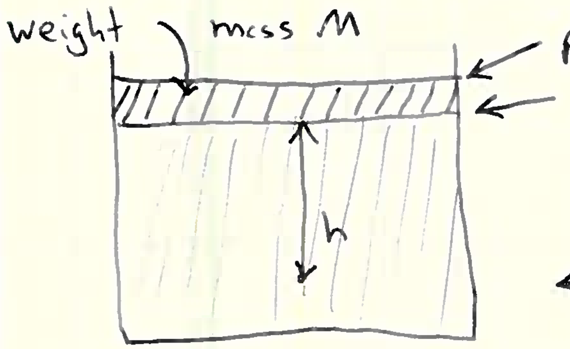


$$p = p_0 + \rho g h$$

e.g. reading tire pressure.

Pascal's principle :

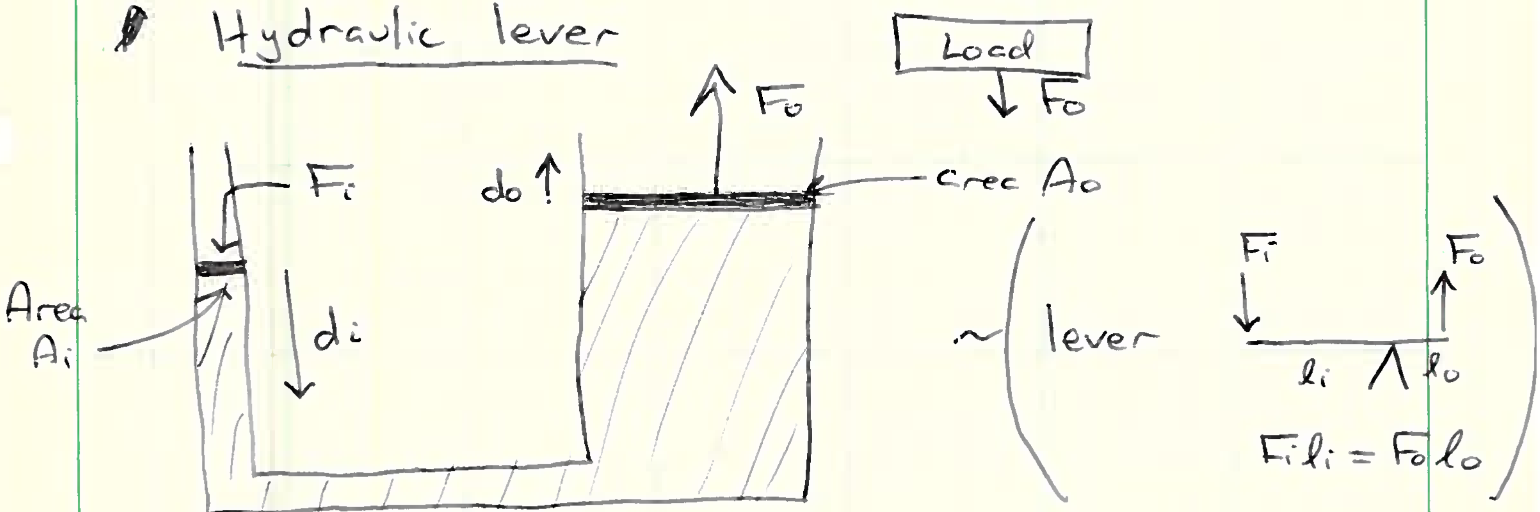
Additional pressure Δp_{ext} exerted on enclosed fluid (incompressible) $\longrightarrow$ Δp_{ext} added to pressure everywhere in fluid.



$\Delta p = Mg/A$

$\leftarrow \; p = p_0 + \Delta p + \rho g h$

Hydraulic lever



$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o} \qquad F_o = F_i \, \frac{A_o}{A_i}$$

magnified for $A_o > A_i$

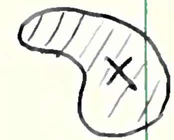
Incompressibility : displacement $d_i A_i = d_o A_o$ (volumes same)

$d_o = d_i A_i / A_o$

Work done on left $\quad F_i d_i = F_o d_o =$ work done by right.

no energy lost (to compression).

Archimedes' Principle: Pressure exerts force $d\vec{F} = -p\,\hat{n}\,dA$ on any area element. $\hat{n}$ = outward nor

Consider object X inside of fluid .

Total force due to pressure = "buoyancy force"

$$\vec{F}_b = -\int_{S_x = \partial V_x} p\,\hat{n}\,dA = -\int_{V_x} \vec{\nabla} p\,dV$$

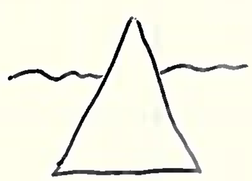
S_x: surface of x V_x: volume of X

But in equilibrium $\vec{\nabla} p = -\rho_{liquid}\, g\, \hat{z}$

so $\vec{F}_b = \hat{z}\, g \int_{V_x} dV \rho_{liquid} = \hat{z} \cdot \left(\text{weight of displaced fluid}\right)$

$$\vec{F}_{total} = \vec{F}_b - M_x g \hat{z} = \left(M_{\substack{\text{displaced}\\\text{fluid}}} - M_{object}\right) g \hat{z}$$

Eg Iceberg



$\rho_{ice} = 917\ \text{kg/m}^3$ @ $0°C$

$\rho_{\substack{sea\\water}} = 1024\ \text{kg/m}^3$ @ $0°C$

$F_b = g V_{sub} \rho_{water}$, V_{sub} = submerged volume

Weight of iceberg: $W = g V_{total} \rho_{ice} = F_b$

$\Rightarrow V_{sub} \rho_{water} = V_{total} \rho_{ice}$. Fraction above water:

$$(V_{total} - V_{sub}) / V_{total} = 1 - \frac{\rho_{ice}}{\rho_{H_2O}} \approx 10\%$$

Hydrodynamics

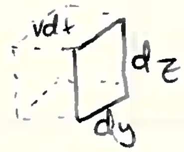
$$p(\vec{r},t)\ ,\quad \rho(\vec{r},t)\ ,\quad \vec{v}(\vec{r},t)$$

eqn of continuity :  mass in element time t : $\rho(\vec{r},t)\,d$

Δ(mass) over time dt : $\dfrac{\partial\rho}{\partial t}\,dV\,dt = \Delta m$

= mass which flows into region (conservation)

$= -\nabla\cdot(\rho\vec{v})\,dV\,dt$

e.g. along x axis 

amount $\rho V_x(x)\,dy\,dz\,dt$ flows in at x

& amount $\left[(\rho V_x)(x+dx) = \rho V_x(x) + \dfrac{\partial(\rho V_x)}{\partial x}\right] dy\,dz\,dt$

flows out at $x+dx$.

so $$\boxed{\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0}$$

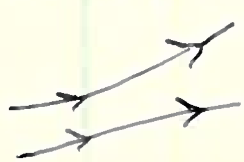
Here take $\rho = \text{const.}$: incompressible

above $\Longrightarrow$ $\nabla\cdot\vec{v} = 0$.

Steady flow : $\vec{v} = \vec{v}(\vec{r})$ no t dep.

non viscous : no friction or drag forces.

Stream lines : tangent to fluid velocity = actual path of fluid particles.

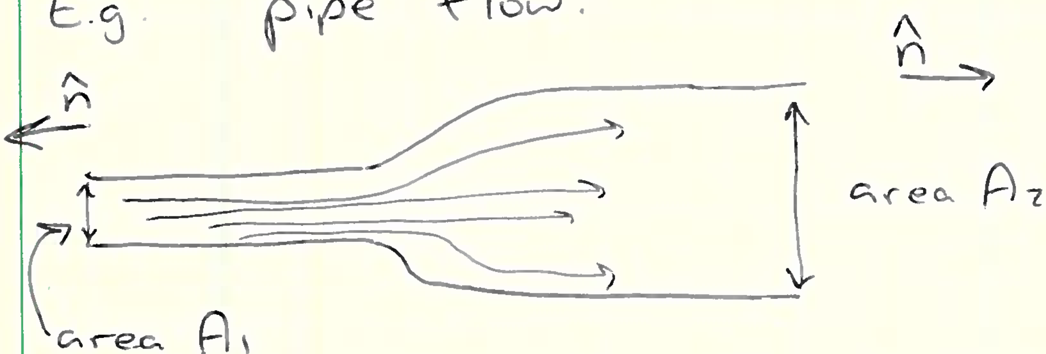


Integrate $\vec{\nabla}\cdot\vec{v} = 0$ over a volume V

$$0 = \int_V dV\, \vec{\nabla}\cdot\vec{v} = \int_{\partial V} \hat{n}\cdot\vec{v}\, dA$$

$\hat{n}$: normal to surface element dA

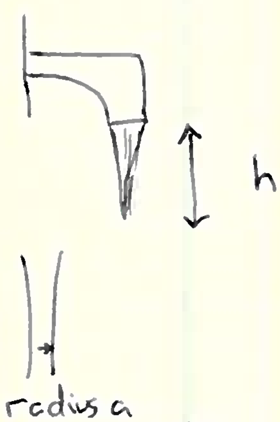
E.g. pipe flow:



$$0 = -V_1 A_1 + V_2 A_2 \quad \Longrightarrow \quad V_1 A_1 = V_2 A_2$$

(larger area $\rightarrow$ smaller velocity)

Faucet



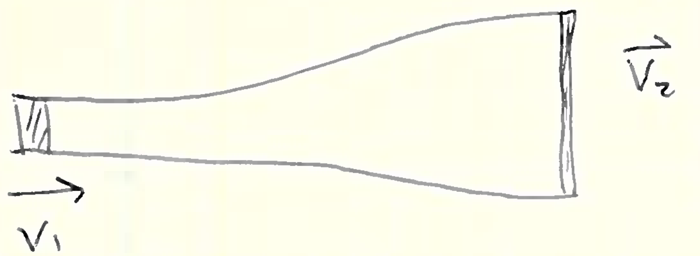
$$V^2(h) = V_0^2 + 2gh$$

$$\pi a^2(h) V(h) = \pi a^2(0) V_0$$

$$\Longrightarrow \quad a(h) = a_0 \Big/ \left(1 + \frac{2gh}{V_0}\right)^{1/4}$$

Bernoulli's Equation : Energy conservation

Consider flow along streamline flow tube



Over time dt,

Work done <u>on</u> fluid inside at end 1 $= p_1 A_1 V_1 dt$

Work done <u>by</u> fluid at end 2 $= p_2 A_2 V_2 dt$.

Net work done on fluid inside :

$$dW = p_1 A_1 V_1 dt - p_2 A_2 V_2 dt$$

continuity $\Rightarrow$ $A_1 V_1 = A_2 V_2$ so $dW = (p_1 - p_2) A_1 V_1 dt$

Kinetic energy changes by $dK = \frac{1}{2}(\rho A_2 V_2 dt) V_2^2 - \frac{1}{2}(\rho A_1 V_1 dt) V_1^2 = \frac{1}{2}\rho (V_2^2 - V_1^2) A_1 V_1 dt$.

Potential energy changes by: $dU = (\rho A_2 V_2 dt) g h_2 - (\rho A_1 V_1 dt) g h_1 = \rho g (h_2 - h_1) A_1 V_1 dt$.

Energy conservation : $dW = dK + dU \Rightarrow$

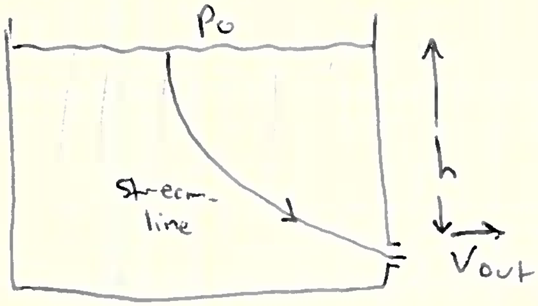
$$p_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2$$

i.e. $\boxed{p + \rho g h + \frac{1}{2}\rho v^2 = \text{constant.}}$

For $h =$ const., larger velocity $\longrightarrow$ lower pressure

e.g. blow between 2 pieces of paper.
They come together.

Eg: 1) Large tank with small hole



Large $\rightarrow$ $V_{top} \approx 0$

(@top): $P_0 + \rho g h = P_0 + \frac{1}{2}\rho V_{out}^2$ (@hole)

$\Rightarrow V_{out} = \sqrt{2gh}$ same as for object dropped from height h.

more generally: velocity V_{top} with $V_t A_t = V_0 A_0$

$$\rho g h + \frac{1}{2}\rho V_t^2 = \frac{1}{2}\rho V_0^2$$

$$V_0^2\left(1 - \left(\frac{A_0}{A_t}\right)^2\right) = 2gh \Rightarrow V_0 = \sqrt{2gh}\Big/\sqrt{1 - \frac{A_0^2}{A_t^2}}$$