# Waves II

Sound waves: longitudinal displacement pressure disturbance.

Wave eqn $\nabla^2 s = \frac{1}{v^2}\frac{\partial^2 s}{\partial t^2}$

plane waves (waves in pipes): $s = A\cos(k(x-vt)+\varphi_0)$

spherical waves: $s = \frac{A}{r}\cos(k(r-vt)+\varphi_0)$

Consider 1st plane waves (focus on 1d)

$P = p_0 + p_e$ pressure

e.g. $p_0 = 1$ atm $p_e = 2\times10^{-7}$ atm

≈ pressure of my voice on your ear.

$\frac{F}{Vol} = \frac{m}{Vol} a$ : $-\frac{\partial p_e}{\partial x} = \rho_0 \frac{\partial^2 s}{\partial t^2}$

$p_e = -B\frac{\partial s}{\partial x}$ B: bulk modulus

$\Delta p = -B\frac{\Delta V}{V}$ ($B=\infty$ for incompressible fluid)

$$\therefore \quad \frac{\partial^2 s}{\partial x^2} = \frac{\rho_0}{B}\frac{\partial^2 s}{\partial t^2} = \frac{1}{V^2}\frac{\partial^2 s}{\partial t^2}$$

so $V_{sound} = \sqrt{B/\rho_0}$ (note relativity $\Rightarrow$ $\therefore \sqrt{\frac{B}{\rho_0}} < c_{light}$)

$V_{H_2O} \rightarrow (20°C) = 1482$ m/s

$V_{air}$ (20°C) $= 343$ m/s

$V_{Al} = 6420$ m/s

$V_{granite} = 6000$ m/s

even though $\rho_{H_2O} = 10^3$ kg/m³

(@ STP) $\rho_{Air} = 1.2$ kg/m³

$B_{H_2O} \gg B_{air}$ (water less compressible than air)

Find B for ideal gas: Take adiabatic compression (realistic because of poor thermal conductivity of air. For better thermal conductivity should use isothermal compression)

$$pV^\gamma = \text{const.} \Rightarrow dp\,V^\gamma + \gamma p V^{\gamma-1} dV = 0$$

$$B = -V\frac{dp}{dV} = \gamma p$$

so in ideal gas $V_{sound} = \sqrt{\frac{\gamma p}{\rho}}$

use $pV = Nk_BT$

$\rho = Nm/V$ m: molecule mass

$$V_{sound} = \sqrt{\frac{\gamma k_B T}{m}}$$

$\gamma = 1 + \frac{2}{f}$

$1 \leq \gamma \leq \frac{5}{3}$

note $V_{sound} \lesssim V_{rms} = \sqrt{\frac{3k_BT}{m}}$

$$s(x,t) = A\cos(k(x-vt)+\varphi_0)$$

$$p_e = -B\frac{\partial s}{\partial x} = BkA\sin(k(x-vt)+\varphi_0)$$

maximum pressure $p_e^{max} = BkA = V^2\rho_0 kA$

$= \underbrace{V\rho}_{const} \underbrace{A\omega}$ → bigger amplitude (louder) or frequency (higher pitch)

→ more pressure on your ear.

largest $p_e^{max}$ before pain or ear damage

28 Pa $\rho = 1.21\ kg/m3$ $f = 10^3 Hz$

$$S_{max} = 28Pa/(343m/s)(\overset{\rho_{air}}{1.21}\ kg/m3)(2\pi)(10^3) = \boxed{1.1\times10^{-5}m}$$

# Intensity & sound level

$$I = \frac{\overline{\text{Power}}}{\text{Area}} \; : \qquad \overline{\frac{dE}{dt}} = \overline{\frac{dK}{dt}} + \overline{\frac{dU}{dt}} = 2\overline{\frac{dK}{dt}}$$

$$\frac{dK}{dt} = \frac{1}{2}\frac{dm}{dt}\left(\frac{\partial s}{\partial t}\right)^2 = \frac{1}{2}\rho\,(\text{Area})\,\frac{dx}{dt}\left(\frac{\partial s}{\partial t}\right)^2$$

$$= \frac{1}{2}\rho\,(\text{Area})\,V_{\text{sound}}\left[\omega^2 s_m^2 \sin^2(kx - \omega t + \varphi)\right]$$

$$\therefore \quad I = \frac{1}{2}\rho V_{\text{sound}}\,\omega^2 s_m^2$$

point source $\quad I = P_s/4\pi r^2 \quad (s_m \sim 1/r)$

## Decibels

$$\beta = (10\,\text{dB})\log_{10}\left(\frac{I}{I_0}\right)$$

$$I_0 = 10^{-12}\,\text{W/m}^2$$

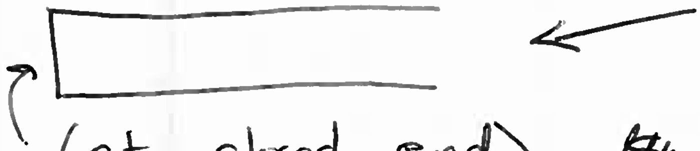
for $I = I_0$, $\beta = 0 \approx$ lower limit of human ear

conversation $\beta = 60$

rock concert $\beta = 110$

pain threshold $\beta = 120$ ← (who 1976 46m from speakers)

Waves in pipes: Solve wave eqn with boundary conditions.



at open end

$P_e = 0 \Rightarrow \frac{\partial s}{\partial x} = 0$

(at closed end
$s = 0$)

e.g.

1) pipe with closed end at $x = 0$ and open one at $x = L$

sol'n $s(x,t) = A \sin(kx) \cos(kvt + \varphi_0)$

where $\cos(kL) = 0 \Rightarrow kL = (n + \frac{1}{2})\pi$

Using $k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{4L}{(2n+1)}$ $(n = 1, 2, \ldots$

etc.

2) pipe with open end at $x = 0$ and open at $x = L$

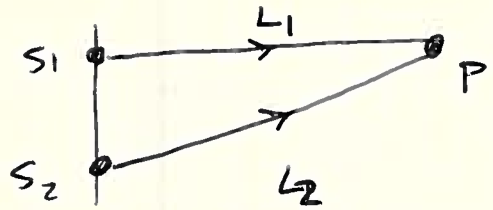
$s(x,t) = A \cos(kx) \cos(kvt + \varphi_0)$

where $\sin(kL) = 0 \Rightarrow kL = n\pi$

i.e. $\lambda_n = \frac{2L}{n}$ $(n = 1, 2, \ldots)$

etc...

# Interference



$$s = A\cos\left(k(L_1 - vt) + \varphi_0\right) + A\cos\left(k(L_2 - vt) + \varphi_0\right)$$

(assume both same amplitude & phase)

use $\cos a + \cos b = 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$

$$s = 2A\cos k\left(\frac{L_2 - L_1}{2}\right)\cos\left(k\left(\frac{L_1+L_2}{2}\right) - \omega t + \varphi_0\right)$$

Constructive int: $k\frac{\Delta L}{2} = n\pi$ $(\cos = \pm 1)$

i.e. (Using $k = \frac{2\pi}{\lambda}$) $\Delta L = n\lambda$

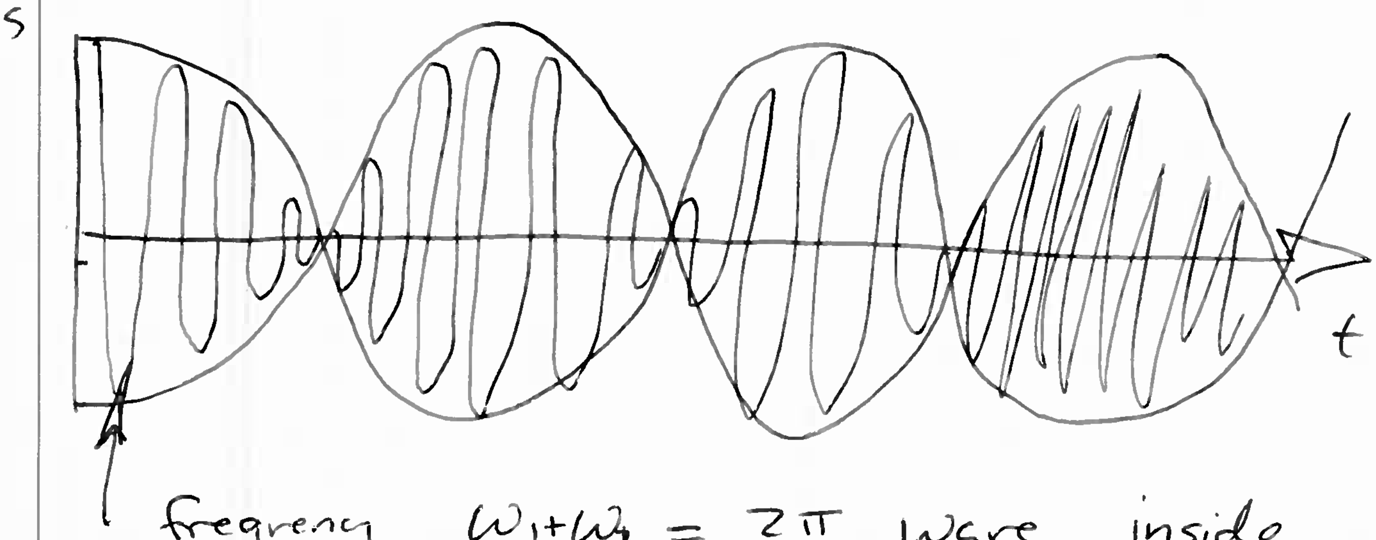
destructive int: $k\frac{\Delta L}{2} = (n + \frac{1}{2})\pi$ $(\cos = 0)$

$$\Rightarrow \Delta L = (n + \tfrac{1}{2})\lambda$$

$s_1 = S_m \cos \omega_1 t \qquad s_2 = S_m \cos \omega_2 t$

$$s = s_1 + s_2 = 2\cos\left(\frac{(\omega_1 - \omega_2)}{2} t\right) \cos\left(\frac{(\omega_1 + \omega_2)}{2} t\right)$$

for $\omega_1 - \omega_2$ small, looks like



fregrency $\frac{\omega_1 + \omega_2}{2} = \frac{2\pi}{T_{wave}}$ wave inside

fregrency $\frac{\omega_1 - \omega_2}{2} = \frac{2\pi}{T_{beat}}$ envelope.

Hear sound get louder, quieter, etc. with period $T_{beat}$. Use to tune instruments.

## Doppler Effect

$$f' = \frac{V - V_D}{V - V_s} f$$

relative to reference frame of air.

for $V_D$ & $V_s \ll V$,

$$f' \approx f\left(1 - \left(\frac{V_D - V_s}{V}\right)\right)$$

For light waves: no preferred frame of reference (Einstein) so different:

$$f' = f\sqrt{\frac{(1 \pm u/c)}{(1 \mp u/c)}}$$ for $u$ = relative speed of source & detector

$$\approx f(1 \pm u/c)$$ for $u \ll c$

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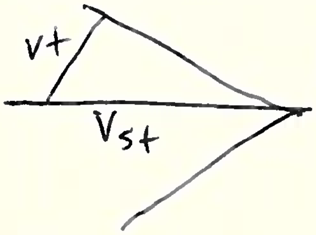
Sonic boom



$$\sin\theta = \frac{Vt}{Vst} = \frac{V}{V_s} = \text{mach \#}$$