

Waves II

Sound waves: longitudinal displacement
pressure disturbance.

Wave eqn $\nabla^2 s = \frac{1}{v^2} \frac{\partial^2 s}{\partial t^2}$

plane waves (waves in pipes): $s = A \cos(k(x-vt) + \phi_0)$

spherical waves: $s = \frac{A}{r} \cos(k(r-vt) + \phi_0)$

Consider 1st plane waves (focus on 1d)

$$P = p_0 + p_e \quad \text{pressure}$$

e.g. $p_0 = 1 \text{ atm}$ $p_e = 2 \times 10^{-7} \text{ atm}$
 \approx pressure of my voice on your ear.

$$\frac{F}{\text{Vol}} = \frac{m}{\text{Vol}} a : \quad - \frac{\partial p_e}{\partial x} = \rho_0 \frac{\partial^2 s}{\partial t^2}$$

$$p_e = -B \frac{\partial s}{\partial x} \quad B: \text{bulk modulus}$$

$$\Delta P = -B \frac{\Delta V}{V} \quad (B = \infty \text{ for incompressible fluid})$$

$$\therefore \frac{\partial^2 s}{\partial x^2} = \frac{\rho_0}{B} \frac{\partial^2 s}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 s}{\partial t^2}$$

$$\text{so } v_{\text{sound}} = \sqrt{B/\rho_0}$$

(note relativity \Rightarrow)

$$\therefore \sqrt{\frac{B}{\rho_0}} < c_{\text{light}}$$

$$v_{\text{H}_2\text{O}} (20^\circ\text{C}) = 1482 \text{ m/s}$$

$$v_{\text{Al}} = 6420 \text{ m/s}$$

$$v_{\text{air}} (20^\circ\text{C}) = 343 \text{ m/s}$$

$$v_{\text{granite}} = 6000 \text{ m/s}$$

even though $\rho_{\text{H}_2\text{O}} = 10^3 \text{ kg/m}^3$

$$\text{(@ STP)} \rho_{\text{air}} = 1.2 \text{ kg/m}^3$$

$B_{\text{H}_2\text{O}} \gg B_{\text{air}}$ (Water ~~more~~ ^{less} compressible than air)

Find B for ideal gas: Take adiabatic

compression (realistic because of poor thermal conductivity of air. For better thermal conductivity should use isothermal compression)

$$pV^\gamma = \text{const.} \Rightarrow dpV^\gamma + \gamma pV^{\gamma-1}dV = 0$$

$$B = -V \frac{dp}{dV} = \gamma p$$

so in ideal gas $V_{\text{sound}} = \sqrt{\frac{\gamma P}{\rho}}$

use $PV = Nk_B T$
 $\rho = Nm/V$ m : molecule mass

$V_{\text{sound}} = \sqrt{\frac{\gamma k_B T}{m}}$ $\gamma = 1 + \frac{2}{f}$
 $1 \leq \gamma \leq \frac{5}{3}$

note $V_{\text{sound}} \lesssim V_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$

$s(x,t) = A \cos(k(x-vt) + \phi_0)$

$p_e = -B \frac{\partial s}{\partial x} = BkA \sin(k(x-vt) + \phi_0)$

Maximum pressure $p_e^{\text{max}} = BkA = v^2 \rho_0 k A$
 $= \underbrace{v^2 \rho_0}_{\text{const}} \underbrace{A \omega}_{\text{bigger amplitude (louder) or frequency (higher pitch)}}$
 \rightarrow more pressure on your ear.

largest p_e^{max} before pain or ear damage
 28 Pa $\rho = 1.21 \text{ kg/m}^3$ $f = 10^3 \text{ Hz}$

$S_{\text{max}} = \frac{28 \text{ Pa}}{(343 \text{ m/s}) (1.21 \text{ kg/m}^3) (2\pi) (10^3)} = 1.01 \times 10^{-5} \text{ m}$

500 SHEETS, FILLER, 5 SQUARE
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 100 RECYCLED WHITE, 5 SQUARE
 200 RECYCLED WHITE, 5 SQUARE
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Intensity & sound level

$$I = \frac{\text{Power}}{\text{Area}} : \quad \frac{dE}{dt} = \frac{dK}{dt} + \frac{dU}{dt} = 2 \frac{dK}{dt}$$

$$\frac{dK}{dt} = \frac{1}{2} \frac{dm}{dt} \left(\frac{\partial s}{\partial t} \right)^2 = \frac{1}{2} \rho (\text{Area}) \frac{dx}{dt} \left(\frac{\partial s}{\partial t} \right)^2$$

$$= \frac{1}{2} \rho (\text{Area}) v_{\text{sound}} \left[\omega^2 s_m^2 \sin^2(kx - \omega t + \phi) \right]$$

$$\therefore I = \frac{1}{2} \rho v_{\text{sound}} \omega^2 s_m^2$$

point source $I = P_s / 4\pi r^2$ ($s_m \sim 1/r$)

Decibels

$$\beta = (10 \text{ dB}) \log_{10} \left(\frac{I}{I_0} \right)$$

$$I_0 = 10^{-12} \text{ W/m}^2$$

for $I = I_0$, $\beta = 0 \approx$ lower limit of human ear

conversation $\beta = 60$

rock concert $\beta = 110$

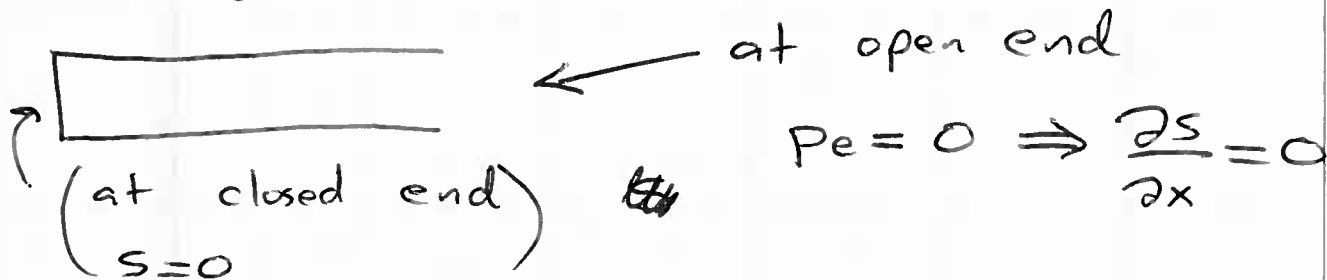
pain threshold $\beta = 120$

← (who 1976)
46m from speakers

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Waves in pipes: Solve wave eqn with boundary conditions.



e.g.

1) Pipe with closed end at $x=0$ and open one at $x=L$

sol'n $s(x,t) = A \sin(kx) \cos(kvt + \phi_0)$

where $\cos(kL) = 0 \Rightarrow kL = (n + \frac{1}{2})\pi$

Using $k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{4L}{(2n+1)} (n=1,2,...)$



etc.

2) pipe with open end at $x=0$ and open at $x=L$

$s(x,t) = A \cos(kx) \cos(kvt + \phi_0)$

where $\sin(kL) = 0 \Rightarrow kL = n\pi$

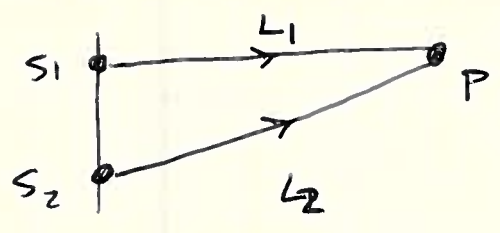
i.e. $\lambda_n = \frac{2L}{n} (n=1,2,...)$

etc...

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Made in U.S.A.



Interference



$$S = A \cos(k(L_1 - vt) + \phi_0) + A \cos(k(L_2 - vt) + \phi_0)$$

(assume both same amplitude & phase)

use $\cos a + \cos b = 2 \cos\left(\frac{a-b}{2}\right) \cos\left(\frac{a+b}{2}\right)$

$$S = 2A \cos k\left(\frac{L_2 - L_1}{2}\right) \cos\left(k\left(\frac{L_1 + L_2}{2}\right) - \omega t + \phi_0\right)$$

Constructive int: $k \frac{\Delta L}{2} = n\pi$ (cos = ±1)

i.e. (Using $k = \frac{2\pi}{\lambda}$) $\Delta L = n\lambda$

destructive int: $k \frac{\Delta L}{2} = (n + \frac{1}{2})\pi$ (cos = 0)

$$\Rightarrow \Delta L = (n + \frac{1}{2})\lambda$$

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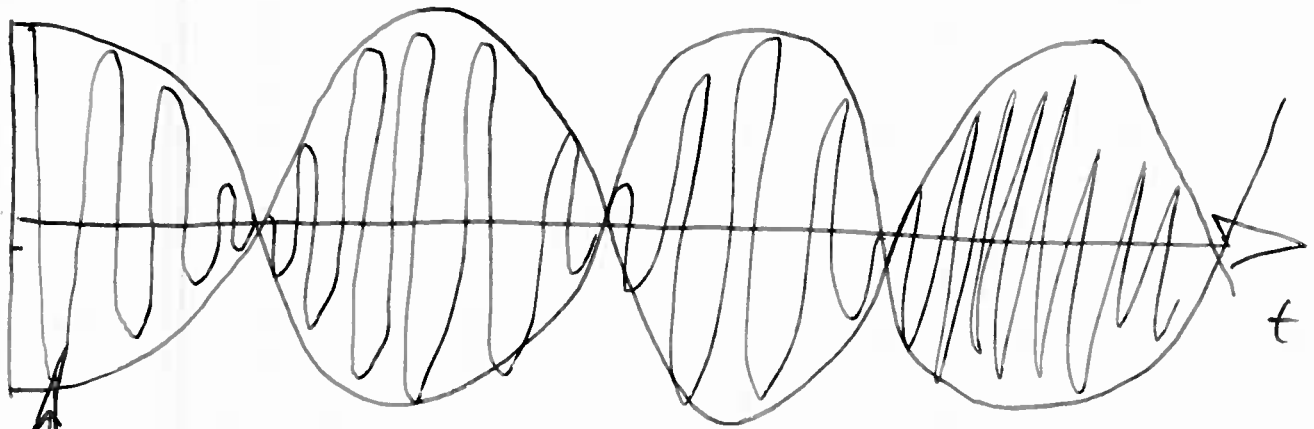


$$S_1 = S_m \cos \omega_1 t$$

$$S_2 = S_m \cos \omega_2 t$$

$$S = S_1 + S_2 = 2 \cos \left(\frac{\omega_1 - \omega_2}{2} t \right) \cos \left(\frac{\omega_1 + \omega_2}{2} t \right)$$

For $\omega_1 - \omega_2$ small, looks like



frequency $\frac{\omega_1 + \omega_2}{2} = \frac{2\pi}{T_{\text{wave}}}$ wave inside

frequency $\frac{\omega_1 - \omega_2}{2} = \frac{2\pi}{T_{\text{beat}}}$ envelope.

Hear sound get louder, quieter, etc.

with period T_{beat} . Use to

tune instruments.

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Doppler Effect



$$f' = \frac{v - v_D}{v - v_s} f$$

relative to reference frame of air.

for $v_D \text{ \& } v_s \ll v$,

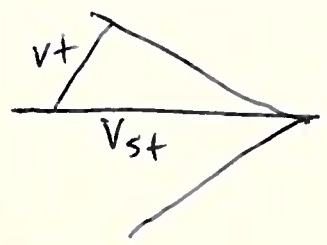
$$f' \approx f \left(1 - \frac{v_D - v_s}{v} \right)$$

For light waves: ^{preferred} no 1 frame of reference (Einstein) so different:

$$f' = f \frac{\sqrt{1 \pm u/c}}{\sqrt{1 \mp u/c}} \quad \text{for } u = \text{relative speed of source \& detector}$$

$$\approx f(1 \pm u/c) \quad \text{for } u \ll c$$

Sonic boom



$$\sin \theta = \frac{vf}{v_{st}} = \frac{v}{v_s} = \text{Mach \#}$$