10/28 Lecture outline

$\star$ **Reading for today's lecture: Coleman lecture notes pages 103-120 (skip parts about counterterms for now).**

• Last time: Feynman rules! Each vertex gets $(-ig)(2\pi)^4\delta^4(p_{total\ in})$, each internal line gets $\int \frac{d^4k}{(2\pi)^4} D_F(k^2)$, where D_F is the propagator, e.g. $D_F(k^2) = \frac{i}{k^2 - m^2 + i\epsilon}$. Result is $\langle f|(S-1)|i\rangle$, so divide by $(2\pi)^4\delta^4(p_F - p_I)$ to get $i\mathcal{A}_{fi}$.

If the diagram has no loops, the momentum conserving delta functions fix all internal momenta in terms of the external ones. When the diagram has $L \neq 0$ loops, the procedure above yields integrals over the internal momenta of the loops. (Note that if a diagram has I internal lines and V vertices, then there are I momentum integrals, and V momentum conserving delta functions; one of these becomes overall momentum conservation, so there are $L = I - (V-1)$ momentum integrals left to do, and L is the number of loops in the diagram.) Any loop momentum integrals require renormalization, which we'll discuss later (next quarter), so for now we'll just consider "tree-level" contributions, associated with diagrams without loops, $L = 0$.

Draw some diagram examples, noting that $L = I - (V-1)$.

• Last time, we had some examples of $2 \to 2$ processes.

(1) $N + N \to N + N$, to $\mathcal{O}(g^2)$

$$i(-ig)^2\left[\frac{1}{(p_1 - p_1')^2 - \mu^2} + \frac{1}{(p_1 - p_2')^2 - \mu^2}\right](2\pi)^4\delta^{(4)}(p_1 + p_2 - p_1' - p_2').$$

(2) $N(p_1) + \bar{N}(p_2) \to N(p_1') + \bar{N}(p_2')$ has

$$i\mathcal{A} = (-ig)^2\left(\frac{i}{(p_1 - p_1') - \mu^2} + \frac{i}{(p_1 + p_2) - \mu^2}\right).$$

(3) $N(p_1) + \bar{N}(p_2) \to \phi(p_1')\phi(p_2')$ has

$$i\mathcal{A} = (-ig)^2\left(\frac{i}{(p_1 - p_1') - m^2} + \frac{i}{(p_1 - p_2') - m^2}\right).$$

(4) $N(p_1) + \phi(p_2) \to N(p_1') + \phi(p_2')$ has

$$i\mathcal{A} = (-ig)^2\left(\frac{i}{(p_1 - p_2') - m^2} + \frac{i}{(p_1 + p_2) - m^2}\right).$$

Note: the $1/2!$ from expanding $e^{-i\int d^4x \mathcal{H}_I(x)}$ is cancelled by a factor of 2 from exchanging the two vertices.

• Mandelstam variables. $s = (p_1+p_2)^2$, $t = (p_1-p_1')^2$, $u = (p_1-p_2')^2$, with $s+t+u = 4m^2$ (more generally, $s+t+u = \sum_{i=1}^4 m_i^2$). In CM, $s = 4E^2$, $t = -2p^2(1-\cos\theta)$, and $u = -2p^2(1+\cos\theta)$.

• Crossing symmetry, CPT. Write $1 + 2 \to \bar{3} + \bar{4}$. Take all momenta incoming, $\mathcal{A}(p_1, p_2, p_3, p_4)$, with $p_1 + p_2 + p_3 + p_4 = 0$ and use $s = (p_1+p_2)^2$, $t = (p_1+p_3)^2$ and $u = (p_1+p_4)^2$. Note $s+t+u = \sum_{n=1}^4 m_n^2$. The process $1+2 \to \bar{3} + \bar{4}$ is kinematically allowed for $s > 4m^2$, $t < 0$, $u < 0$. If instead $u > 4m^2$, it's the process $1+3 \to \bar{2} + \bar{4}$.

• Scattering by ϕ exchange leads to an attractive Yukawa potential. This was Yukawa's original goal, to explain the attraction between nucleons. Indeed, the t-channel term in e.g. the above $N+N$ scattering amplitude gives, upon using $(p_1-p_1')^2 - \mu^2 = -(|\vec{p}_1 - \vec{p}_1'|^2 + \mu^2)$, and the Born approximation[1] in NRQM, $\mathcal{A}_{NR} = \int d^3\vec{r} e^{-i(\vec{p}'-\vec{p})\cdot\vec{r}} V(\vec{r})$, the attractive Yukawa potential

$$V(r) = \int \frac{d^3q}{(2\pi)^3} \frac{-(g/2m)^2 e^{i\vec{q}\cdot\vec{r}}}{|\vec{q}|^2+\mu^2} = -\frac{(g/2m)^2}{4\pi r} e^{-\mu r}.$$

(The $1/(2m)^2$ is because we normalized the relativistic states with the extra factor of $2E \approx 2m$ as compared with standard nonrelativistic normalization[2]. This gives Yukawa's explanation of the attraction between nucleons, from meson exchange. The u-channel term is an exchange potential interaction, which exchanges the positions of the two identical particles in addition to giving a potential. For angular momentum ℓ in a partial-wave expansion, the exchange term differs from the direct one by a factor of $(-1)^\ell$.

[1] Max Born, in QM, or Lord Rayleigh classically: $\frac{d\sigma}{d\Omega} \sim |U(\vec{q})|^2$

[2] This is clear on dimensional grounds, since $[g] \sim m$. More generally, write $a(p) = \sqrt{2E}\widehat{a}(p)$ and $\mathcal{A} = \prod_i \sqrt{2E_i} \prod_f \sqrt{2E_f} \widehat{\mathcal{A}}$.