Physics 105a, Ken Intriligator lecture 5, October 12, 2017

• Recap from end of last time: example of an L, R circuit, driven by source $V(t) = A\int e^{i\omega t}d\omega/2\pi$; this source corresponds to a voltage spike at time $t=0$. Find $I(t) = A\int (R+i\omega L)^{-1}d\omega/2\pi$. Discuss where to close the contour and get $I(t<0)=0$ and $I(t>0) = (A/L)e^{-Rt/L}$. Makes sense.

• Residues and poles of $\pi/\sin(\pi z)$ and $\pi\cos(\pi z)/\sin(\pi z)$ and applications of Cauchy's theorem to evaluate some sums, $\sum_{n=1}^{\infty} f(n)$. Examples in the mathematica notebook.

• Gamma function $\Gamma(z)$; give integral definition and type it into mathematica, $\Gamma(z+1) = z\Gamma(z)$ and relation to factorial. Poles at $x=0$ and negative integers. Check with mathematica. Also $\Gamma(z)\Gamma(1-z) = \pi/\sin(\pi z)$.

• **Maybe later:** Gaussian integral, including in multi-dimensions. Normalization of the normal distribution. Relation to spherical integrals and solid angles.

• **Later:** Example of forced SHO with $F(t) = F\cos(\omega t)$ and the particular solution for both $\omega \neq \omega_0$ and $\omega = \omega_0$.

• Suppose that we want to solve the ODE $\frac{d^2x}{dt^2} = f(x,\dot{x})$, where f is some given function, e.g. $f = -\omega_0^2 x - \gamma v$ for the case of a damped SHO. Note that we are here taking $f(x,\dot{x})$ to not depend explicitly on t. Plot $(x,\dot{x},t)$ curve, so $(dx, dv, dt) = dt(v, f, 1)$ is the tangent vector. Project to the (x,v) plane, and plot (v,f) to give the tangent vector field. For example, for the SHO, $f=-x$ so the tangent vector field in the (x,v) plane is $(v,-x)$, i.e. the phase space motion is a circle. Discuss example in cell 1.6 of Chapter1.nb. It is often useful to use p instead of $\dot{x}$ (in simple cases, this is just a rescaling as $p = m\dot{x}$). Plot phase space motion for the solution of the undamped SHO vs the damped SHO.

• Non- dissipative systems have conserved energy and the flow in the (x,v) plane has zero divergence, i.e. $\partial_x \dot{x} + \partial_v \dot{v} = 0$. Hence the area in phase space is constant in time. For e.g. the SHO both terms are zero, whereas for the damped SHO the 2nd term is non-zero.

• **Later:** Hamiltonian flows: $H(x,p,t)$ with $\dot{x} = \partial_p H$ and $\dot{p} = -\partial_x H$. Discuss $\dot{H}$ vs $\partial_t H$ and show that $\dot{H} = 0$ if $\partial_t H = 0$: this is conservation of energy if the system does not explicitly depend on t. Get $dH = 0$, so the flow is along surfaces of constant H. You will learn more about this in physics 110.

• Following Dubin 1.4.3, discuss Euler's method for numerical solutions of differential equations. First consider $\dot{v} = f(t,v)$ e.g. for $f(t,v) = t - v$, with $v(0) = 0$. The exact solution is $v(t) = ce^{-t} + t - 1$, and the initial condition gives $c = 0$. Approximate via making time a lattice, with $t_n = n\Delta t$ and $v(t_n) = v_n$ such that $v_n - v_{n-1} = \Delta t f[t_{n-1}, v_{n-1}]$, which is a recursion relation for the v_n. We will use mathematica to iterate this.