

Physics 220, Homework 1, Due April 5.

1. Recall that the “order” of a group G , written in class as $|G|$, is the number of its elements. The “order” of an element $g \in G$ is the *smallest* integer n such that $g^n = e$. Suppose someone tells you that they found an order 6 group with no order 6 elements and no order 3 elements. Show that they’re mistaken.
2. Let $a = (1234)(5678)$ and $b = (1537)(2846)$ be two elements of the permutation group S_8 . (Recall the notation from class: a takes the objects in positions 1,2,3,4 and moves the object in position 1 to position 2 etc.) Take products of these two elements until you fill out (or “generate”) a complete group. Show that this group has order 8 and that it’s isomorphic to the quaternion group, with the 8 elements $\{1, -1, i, -i, j, -j, k, -k\}$ and multiplication rule $i^2 = j^2 = k^2 = -1$, $ij = k$, $jk = i$, $ki = j$. (Here 1 is the identity element and the other multiplication rules are obvious things like $(-1)(-1) = 1$, $(-1)(i) = -i$ etc. From these it follows e.g. that $ji = j(jk) = (jj)k = -k$, so this is a non-Abelian group.) “Isomorphic” means that you can find a dictionary relating each group element, e.g. a maps to one of the quaternion elements, which respects the group multiplication table; so you’re being asked to find such a dictionary or map. Also, find the conjugacy classes of this group.
3. Georgi 1A.
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