Physics 220, Homework 2, Due April 22 (or 26 if needed).

1. Last week you worked out the conjugacy classes of the quaternion group: $K_0 = \{e\}$, $K_1 = \{-1\}$, $K_2 = \{i, -i\}$, $K_3 = \{j, -j\}$, $K_4 = \{k, -k\}$.

   a) Explicitly construct all possible 1d representations of the group.

   b) Construct the character table for the group, using the character orthogonality etc. relations to fill in the entries for the $n_a > 1$ dimensional irrep(s). (The entries for the 1d irreps follow from the results of part (a).)

2. The tensor product of two irreps can be decomposed in terms of irreps: $D_a \otimes D_b = \oplus_c N_{ab}^c D_c$. The character satisfies $\chi_{D_a \otimes D_b}(g) = \chi_{D_a}(g)\chi_{D_b}(g)$. Using this and your results from the previous problem, write out all of the tensor product decompositions for the quaternion group ($D_a \otimes D_b =?$ for every irrep $a$ and $b$).

3. Georgi 1E. Include only proper rotations, no inversions. Let's call this group $T$; $T$ is an (invariant) subgroup of $S_4$, corresponding to the permutations of the 4 vertices under rotation. So write all of the $T$ group elements out in the $S_4$ notation as in sect. 1.19 of Georgi, e.g. $e = (1)(2)(3)(4) \in T$, and $(123)(4) \in T$, (whereas the $S_4$ element $(1234)$ is not in $T$). You needn't bother writing out the full group multiplication table; just the group elements, conjugacy classes, and character table. Hints: As a check that you got all of the group elements, remember that the subgroup satisfies $|T| = |S_4|/m$ for some integer $m$. Also, be careful with the conjugacy classes: what was a single conjugacy class in $S_4$ can split into more than one conjugacy class in $T$.

4. For the group $T$ of the previous problem, work out all of the tensor product decompositions, among all irreps $D_a \otimes D_b =?$ (using the characters).