Physics 220, Spring 2004, Homework 4, fun with O, Due May 20

1. Label a rotation by angle θ about some axis $\widehat{n}$ as $R_{\widehat{n}}(\theta)$. Conjugating by another group element, $R_{\widehat{n}_1}(\theta_1)R_{\widehat{n}}(\theta)R_{\widehat{n}_1}(\theta_1)^{-1} = R_{\widehat{n}_2}(\theta)$, where $\widehat{n}_2 = R_{\widehat{n}_1}(\theta_1)\widehat{n}$ is what we get by rotating the axis $\widehat{n}$. I.e. all rotations by some angle θ, around whatever axis, are in the same conjugacy class. The character of all group elements in this conjugacy class is $\chi_j(\theta) = \mathrm{Tr}D(R_{\widehat{z}}(\theta) = \sum_{m=-j}^{+j} e^{im\theta} = \sin[(j+\frac{1}{2})\theta]/\sin(\theta/2)$.

   Suppose that an electron in an atom is in a $j = 2$ state ("d-state"), with all five energy levels initially degenerate. Now a perturbation breaks the full rotation symmetry to the O octahedral subgroup. To find how the $j = 2$ energy levels split, find how the 5d, $j = 2$ irrep. of the full rotation group decomposes into a sum of irreps of the O subgroup: $D_{j=2} = \oplus(O\ irreps)$. To do this, write again the O character table, and include the character $\chi_{j=2}(\theta)$ for the appropriate rotation angles θ corresponding to the various discrete rotations of the O conjugacy classes (e.g. $\theta = 2\pi/3$ for the $2\pi/3$ rotations around the body diagonal axes. Draw a qualitative picture of the splitting of the 5-fold degeneracy. What degeneracies remain?

2. Georgi 3A.

3. Georgi 3B.

4. Georgi 4A.

5. Georgi 4B.