5/24/07 Lecture 15 outline

• A bit more on the harmonic oscillator. Again, $[a, a^\dagger] = 1$ implies:

$$a|n\rangle = \sqrt{n}|n-1\rangle \qquad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle.$$

Explain the names, creation and annihilation operators – and phonons.

•Now for something general. The solution of the S.E. can be written as $|\psi(t)\rangle = \exp(-iHt/\hbar)|\psi(0)\rangle$. Note that the expectation value of an operator O satisfies

$$\langle\psi(t)|O|\psi(t)\rangle = \langle\psi(0)|e^{iHt/\hbar}Oe^{-iHt/\hbar}|\psi(0)\rangle.$$

In the Schrodinger picture, the time dependence comes entirely from that of the bra and ket. Alternatively, in the Heisenberg picture, one pictures the states as fixed, and the operators as varying, as $O(t) = e^{iHt/\hbar}Oe^{-iHt/\hbar}$. Note then that $\frac{d}{dt}O(t) = \frac{1}{i\hbar}[O, H]$, which fits with what we said before (4/24 lecture, here for the case where O has no explicit time dependence).

• In particular, show results for the case of SHO: $\frac{d}{dt}a(t) = \frac{1}{i\hbar}[a, H] = i\omega[a^\dagger a, a] = -i\omega a(t)$, so $a(t) = e^{-i\omega t}a(0)$. Using $a = \sqrt{\frac{m\omega}{2\hbar}}\widehat{x} + i(2m\omega\hbar)^{-1/2}\widehat{p}$, verify that this gives the classical equations $\dot{x} = p/m$ and $\dot{p} = -m\omega^2 x$, but now for time dependent operators.

• Measurement: measure observable corresponding to operator A, get one of A's eigenvalues, α, with probability $|\langle\alpha|\psi\rangle|^2$. The measurement projects $|\psi\rangle \to |\alpha\rangle$. Now measure observable corresponding to operator B, get one of its eigenvalues, β. Explain difference between cases where $[A, B] = 0$ and where $[A, B] \neq 0$.

• Our next topic is QM in 3 space dimensions. An important case is when there is rotation symmetry. Then angular momentum is conserved. We can see that because $[H, \vec{L}] = 0$. So we can simultaneously measure energy and some components of angular momentum.

• But note that different components of the angular momentum don't commute! Angular momentum commutation relations $[L_x, L_y] = i\hbar L_z$ and cyclic permutations. Implies that we can't measure different components simultaneously.