4/12/07 Lecture 4 outline

• Describe interference in terms of individual photons (or electrons or anything). Also, describe outcome from series of polarizing sheets in terms of individual photons. How? Probabilities! Probability amplitude interpretation of electron wave. Instead of particle trajectory, $x(t)$, have probability amplitude, $\psi(x,t)$. Linear superposition of **probability amplitudes**, e.g. 2 slits: $\psi(x,t) = \psi_1(x,t) + \psi_2(x,t)$. The amplitudes add, but the probabilities do not. The probability that particle is found in region from x to $x + dx$ is $P(x,t)dx = |\psi(x,t)|^2 dx$. Probability $P \sim |\psi|^2$ exhibits interference, $P \neq P_1 + P_2$, because of the cross terms, $|\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + \psi_1^*\psi_2 + \psi_2^*\psi_1$. E.g. $\psi_1 = e^{ipx_1/\hbar}$ and $\psi_2 = e^{ipx_2/\hbar}$, with $p = \hbar = h/\lambda$, get constructive interference if $x_1 - x_2 = n\lambda$ and destructive if $x_1 - x_2 = (n + \frac{1}{2})\lambda$, ($n$ is an integer).

• Try to check which slit electron went through, and interference patten goes away.

• Uncertainty principle: $\Delta x \Delta p_x \geq \hbar/2$. E.g. try to resolve the location of an electron better by using light of smaller wavelength. But such light has bigger momentum, leads to greater uncertainty in electrons momentum.

• Free particle of mass m: momentum $\vec{p} = \hbar\vec{k}$ and energy $E = \vec{p}^2/2m = \hbar\omega$. Write probability amplitude as a plane wave

$$\psi(\vec{x},t) \sim e^{i(\vec{p}\cdot\vec{x} - Et)/\hbar}.$$

Note this has $|\psi|^2$ which is independent of position and time: particle's position is completely unknown, special case of Heisenberg uncertainty principle, $\Delta x \Delta p_x \geq \hbar/2$. This is related via $\vec{p} = \hbar\vec{k}$ to a basic property of Fourier transforms, $\Delta x \Delta k \geq \frac{1}{2}$. Get state of position localized in range Δx by a superposition of plane waves, i.e. Fourier transform, with wavenumber range Δk.

• To avoid rewriting too much, write Fourier transform in notation of momentum p, with $p = \hbar k$. Write first for just 1d.

$$\psi(x) = \int_{-\infty}^{\infty} \frac{dp}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}\phi(p), \qquad \phi(p) = \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi\hbar}} e^{-ipx/\hbar}\psi(x).$$

This is the Fourier transform and its inverse. The probability of finding particle in range from x to $x + dx$ is $P(x)dx = |\psi(x)|^2 dx$. The probability to find the particle having momentum in the range from p to $p + dp$ is $P(p)dp = |\phi(p)^2 dp$.

Consider e.g. the case where $\psi(x)$ is a Gaussian, centered at the origin

$$\psi(x) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^{1/2} \exp(-x^2/4\sigma^2) \to |\phi(p)|^2 = \frac{1}{\sqrt{2\pi}\tilde{\sigma}} \exp(-p^2/2\tilde{\sigma}^2),$$

where $\sigma\tilde{\sigma} = \hbar/2$. Note that the above wavefunction examples are properly normalized, to have total probability one:

$$\int_{-\infty}^{\infty} |\psi(x)|^2 = 1, \qquad \int_{-\infty}^{\infty} |\phi(p)|^2 = 1.$$

Find for this example that $\Delta x^2 = \langle x^2 \rangle - \langle x \rangle^2 = \sigma^2$ and $\Delta p^2 = \langle p^2 \rangle - \langle p \rangle^2 = \tilde{\sigma}^2$, so this case saturates the uncertainty principle inequality (it's an equality for the special case of gaussians).