130a Homework 1, due 4/10

1. Consider the harmonic oscillator: a particle of mass m is on a frictionless surface, and connected to a spring. Let x be the displacement of the mass from equilibrium. The spring provides the restoring force $F_x = -\kappa x$.

(a) Write the E.O.M. (equations of motion) and show that a solution is $x_{sol}(t) = \mathrm{Re} A e^{-i\omega t}$ where $\omega = \sqrt{\kappa/m}$.

(b) Write the Hamiltonian $H(x,p) = K.E. + P.E.$ in terms of x and p.

(c) Verify that the Hamilton equations

$$\dot{x} = \frac{\partial H}{\partial p} \qquad \dot{p} = -\frac{\partial H}{\partial q}$$

are satisfied. The above equations are particular cases of what can be written more generally as

$$\frac{df}{dt} = \{f, H\}_{P.B.}$$

where $f = f(x,p)$ is any function of x and p, with no explicit t dependence, and where the Poisson Bracket is defined as

$$\{A, B\}_{P.B.} \equiv \frac{\partial A}{\partial x}\frac{\partial B}{\partial p} - \frac{\partial A}{\partial p}\frac{\partial B}{\partial x}.$$

(d) Pug in the above solution $x_{sol}(t)$ and verify that $H(x_{sol}, p_{sol}) = \frac{1}{2} m\omega^2 A^2$, i.e. a constant independent of time. This happens whenever H does not have explicit t dependence (i.e. it only depends on t implicitly, via x and p). This is a statement of conservation of energy.

(e) Show that $\langle\langle K.E.\rangle\rangle = \langle\langle P.E.\rangle\rangle = \frac{1}{4} m\omega^2 A^2$, where $\langle\langle\cdot\rangle\rangle$ denotes the time-average of the quantity inside over an oscillation period T, i.e. $\langle\langle f\rangle\rangle \equiv \frac{1}{T}\int_0^T dt f(t)$.

2. In class, we found $e(\omega, T)d\omega$ the power per unit area emitted in the frequency range $d\omega$. Substitute $\omega = 2\pi c/\lambda$ and define $\tilde{e}(\lambda, T)d\lambda = e(\omega, T)d\omega$. The function $\tilde{e}(\lambda, T)$ has a maximum at wavelength λ_{peak}. Verify that $\lambda_{peak}T = b$ for some constant b, and write the equations that determines b. This yields Wein's law $\lambda_{peak}T \approx 2.898 \times 10^{-3} Km$.

3. The sun radiates power $P = 4.5 \times 10^{25} W$. What is the radiated power of a star whose radius is a factor of 3 bigger than that of the sun, and whose peak wavelength is a factor of 4 bigger than the peak wavelength of the sun?

4. Consider a spherical cavity of radius 1 meter. How many possible light wave modes are there in the cavity having frequency in the spectrum that's visible to the human eye (wavelengths in the range from $4 \times 10^{-7} m$ and $7 \times 10^{-7} m$)?

5. When light of a certain wavelength λ is incident on a certain metal, the stopping potential for the photoelectron current is found to be $5V$. When light of wavelength 2λ is incident on the same metal, the stopping potential is $1V$. What is the work function of the metal, in units of eV?