130a Homework 6, Due May 22, 2008.

1. Consider the particle of mass m in the infinite potential well. Suppose that

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{3}}|E_{n=1}\rangle + \sqrt{\frac{2}{3}}|E_{n=2}\rangle.$$

(a) Use the fact that $|\psi(t)\rangle = \exp(-iHt/\hbar)|\psi(t=0)\rangle$ to write an expression for $\psi(x,t) = \langle x|\psi(t)\rangle$, as an explicit function of x and t.

(b) Compute $\langle \widehat{x}\rangle$, and $\langle \widehat{p}\rangle$, as a function of time t, in the state $\psi(x,t)$. Is $\langle p\rangle = m\frac{d}{dt}\langle \widehat{x}\rangle$?

2. Shankar 5.2.2 (p. 163).

3. Shankar 5.2.3 (p. 163).

4. Shankar 5.3.4 (p. 167).

5. Shankar 5.4.2a (p. 175).

6. Shankar exercise 11.4.1 (p. 300). Use $|\psi(t)\rangle = \exp(-iHt/\hbar)|\psi(t=0)\rangle$.

7. Consider a particle of mass m, which moves in 1 dimension (the x direction), with potential $V(x) = \frac{1}{2}m\omega^2 x^2$. Suppose that the particle is initially in a state with wavefunction $\psi(x,t=0) = \psi_0(x) \equiv C_0 e^{-\alpha_0 x^2}$.

(a) Determine the correct value of C_0 (in terms of α_0) for this to lead to a properly normalized probability distribution. (A good check: verify that the units work).

(b). Verify that $\psi_0(x)$ is a state of definite energy (a energy eigenvector), provided that α_0 takes a particular value. Determine this value of α_0 (in terms of m, ω, and $\hbar$). Show all your work for full credit.

(c) What is the energy eigenvalue E_0 corresponding to the above state $\psi_0(x)$ (assuming the particular value of α_0 determined above)? Show all your work for full credit.

(d) Suppose that the initial wavefunction is $\psi(x,t=0) = \psi_0(x)$. What is the probability that the particle is found to have momentum in the range from p to $p+dp$?