4/3/08 Lecture 2 outline

• Continue from last time. Experiment with waves and particles (classical). Illustrate interference. Slits S_1 and S_2. With just S_1 open get intensity $I_1 = |\psi_1|^2$, and with just S_2 open get $I_2 = |\psi_2|^2$. Arrival of energy is smooth function of x and t. With both open get $I_{1+2} = |\psi_1 + \psi_2|^2 \neq I_1 + I_2$: interference pattern. Maxima when $\Delta\phi = 2\pi n$, and minima when $\Delta\phi = (2n+1)\pi$, with n integer. At x_{min}, opening extra other slit has the effect of reducing the energy flow there. Now contrast with particles, where classical physics suggests $I_{1+2} = I_1 + I_2$.

• Double-slit experiment with light. Find packets of energy and momentum, and also interference! $E = \hbar\omega$ and $\vec{p} = \hbar\vec{k}$. $\hbar = h/2\pi \approx 10^{-27} ergsec$. Born: $\psi(\vec{r}, t)$ is a probability amplitude, $|\psi(\vec{r}, t)|^2$ gives the probability. de Broglie: matter behaves the same way. E.g. a pellet with mass $1g$ and velocity $1cm/sec$ has $\lambda = 2\pi/k = h/p \approx 10^{-26}cm$. This is too small to notice for objects that we see directly, but becomes important for objects like electrons.

• Now expand a bit on the history.

• An old question: is light a particle or a wave? In 1800 Young's double slit experiment showed interference, suggests light is a wave. Later understood as solution of Maxwell's equations. Double slit, intensity $I \sim |\vec{E}_1 + \vec{E}_2|^2 = 4I_1 \cos^2(k\Delta L/2)$. Also explains Snell's law, lenses, thin film interference, diffraction, diffraction gratings, etc. Visible light has $\lambda \sim 4 - 7 \times 10^{-7}m$.

• But it turns out that this, plus concepts from thermodynamics, leads to a paradox. Also, disagreement with experiments around 1900.

• Blackbody radiation and the UV catastrophe. Picture each $\vec{k}$ mode as a harmonic oscillator, one for each polarization. Let $N(\omega)d\omega$ be the number of wave modes in the frequency range from ω to $\omega + d\omega$. Consider first waves on a string, $\psi(x,t) = A\sin(kx)\cos(\omega t)$, $k = n\pi/L$, $n = 1, 2\ldots$. The number of modes in interval Δk is $L\Delta k/\pi$, so $N(k)dk = Ldk/2\pi$, where the 2 is because the standing wave is a superposition of 2 traveling waves, with k and $-k$. So can replace $\sum_{n=1}^{\infty} \to \int_{-\infty}^{\infty} Ldk/2\pi$. For 3d waves, we replace $\sum_{n_x,n_y,n_z} \to \int Vd^3\vec{k}/(2\pi)^3$. Number of modes in $d^3\vec{k}$ is $Vd^3\vec{k}/(2\pi)^3$. Write in spherical coordinates, use $\omega = ck$, and recall there are 2 polarizations (for light), to get

$$N(\omega)d\omega = \frac{V\omega^2 d\omega}{\pi^2 c^3}.$$

Note that this density diverges for large frequencies.

Each $\vec{k}$ mode of light behaves as a harmonic oscillator. A classical harmonic oscillator, at temperature T, has average energy k_BT, independent of the frequency. The energy density (energy per unit volume of the box) in the range ω to $\omega + d\omega$ would then be $u_{cl}(\omega, T)d\omega = N(\omega)k_BTd\omega/V = k_BT\omega^2 d\omega/\pi^2c^3$. Crazy - we would be cooked! For any $T \neq 0$, would have divergent energy density at large frequencies. UV catastrophe. A paradox in classical physics.

• Planck's fix: assume radiation of frequency ω can only be absorbed or emitted in quantized amounts, given by $E = n\hbar\omega$ for integer n. Gives energy density of the glowing light in the cavity:

$$u(\omega, T) = \left(\frac{\hbar\omega}{e^{\hbar\omega/k_NT} - 1}\right)\left(\frac{\omega^2}{\pi^2c^3}\right).$$

(Planck originally wrote this in terms of $\nu = \omega/2\pi$ and $h = 2\pi\hbar$). For low frequencies, $\hbar\omega \ll k_BT$, the first factor $\to k_BT$, and so $u \to u_{cl}$ in this limit. For high frequencies, the first factor goes to zero as $e^{-\hbar\omega/k_BT}$, avoiding the UV catastrophe.

Knowing the energy density inside the cavity also gives the energy flux through the surface (e.g. if there were a hole). The emitted power per unit area per frequency is related to the above energy density by

$$e(\omega, T) = \frac{1}{4}cu(\omega, T) = \frac{1}{4}c\left(\frac{\hbar\omega}{e^{\hbar\omega/k_NT} - 1}\right)\left(\frac{\omega^2}{\pi^2c^3}\right).$$

This is the famous blackbody spectrum of radiated power. It appears everywhere in Nature, e.g. the radiated power of a star (or the entire universe) is given by this formula. Fits beautifully all the experimentally observed data, for $h = 2\pi\hbar = 6.6261 \times 10^{-34} J \cdot s$. (Explain the 1/4: light has velocity c, but only component perpendicular to an area element counts. Let the normal to the area element be $\hat{z}$ and use spherical coordinates, with light direction given by θ, ϕ and our desired flux is then $e = cu\langle\cos\theta\rangle$. Here we average $\langle\cos\theta\rangle = \int_{hemi}\cos\theta d\Omega/\int_{hemi} d\Omega$, where $d\Omega = \sin\theta d\theta d\phi$ and *hemi* is because only θ between 0 and $\pi/2$ leads to a flux out of the area element (for θ between $\pi/2$ and π, the flux is inward); this gives $\langle cos\theta\rangle = 1/4$.)

The above expression is power radiated per area per frequency range. Integrating it over all frequency, get the Stephan-Boltzmann result for the total *power per unit area*: $e_{total}(T) = \int_0^\infty e(\omega, T)d\nu = \sigma T^4$, with $\sigma = 2\pi^5k_B^4/15c^2h^3$. These relations are very useful in astrophysics and cosmology.