4/3/09 Lecture outline

⋆ Reading: Zwiebach chapters 1 and 2.

• Physics develops in patches, which sometimes seem to clash; this can lead to major conceptual advances. Examples: • Maxwell theory of electromagnetism vs. Newtonian mechanics: "what do you see in the mirror if your train moves at the speed of light?" led to special relativity. Physics the same in all inertial references frames, leads to space and time getting mixed up in different inertial frames. Some bizarre consequences, like the ultimate speed limit of light, length contraction, time dilation, twin paradox.

The speed of light is a fundamental quantity but perhaps better viewed as merely a conversion factor, relating space and time. Indeed, we can – and will – choose units where $c = 1$.

Special relativity was a radical departure from previous physics. Many physicists initially thought it was wrong. Completely counterintuitive, since we don't have everyday intuition about speeds $v \sim c$. (Fun: read Mr. Tomkins in Wonderland, by George Gamow!)

• Maxwell theory vs. thermodynamics: blackbody radiation UV catastrophe led to quantum mechanics. $E = \hbar\omega$ and $\vec{p} = \hbar k$, which we can write in relativistic notation as $p^\mu = \hbar k^\mu$. $\Delta p \delta x \geq \hbar/2$, quantum fuzziness, and fundamentally probabilistic ("God plays dice?"). Leads to bizarre consequences, e.g. matter probability waves and interference. Schrodinger's cat, etc..

Planck's constant is a fundamental quantity but, actually, it can also be viewed as merely a conversion factor; we can – and will – choose units where $\hbar = 1$.

Quantum mechanics was a radical departure from previous physics. Many physicists, including Einstein, initially thought it was wrong. Completely counterintuitive, since we don't have everyday intuition about systems where $S \sim \hbar$ (our usual intuition is about the limit $S \gg \hbar$). (More fun: read Mr. Tomkins in Wonderland, by George Gamow!)

• Special relativity vs Newtonian Mechanics. Led Einstein to general relativity, which posits a simple idea, the Principle of equivalence: a local (small) observer in free-fall frame, over short time, feels no gravity – indistinguishable from inertial frame in empty space. Many surprising consequences. Clocks run slower in stronger gravitational fields, black holes, etc. Dynamical metric $g_{\mu\nu}(x)$, Einstein's equations, black holes. Lot's of counter-intuitive stuff. The pinnacle of classical physics.

• Special relativity vs quantum mechanics. Led to quantum field theory (QFT) . All matter and interactions arise as fluctuations of fields. Light consists of photons, which are fluctuations of the field A_μ. Electrons and quarks are fluctuations of the electron and quark fields, etc. Such fluctuations can be created and/or destroyed, and can even exist only temporarily (virtually). Bizarre consequences, e.g. the existence of anti-matter, predicted by Dirac in 1928 and discovered by Anderson (at Caltech) in 1932.

Quantum field theory is one of humankind's most accurately tested theories! E.g. the magnetic moment of the electron, can compute using theory and compare with experiment, get better than 1 part in 10^{12} agreement. A particular model, the "Standard Model", describes the strong, weak, electromagnetic interactions. What about gravity? Gravity is very different from other forces. Other forces carried by spin 1 object: photon, $W^{\pm}$, Z, gluons. Gravity is carried by a spin 2 object: the graviton $\sim \delta g_{\mu\nu}$. Spin 2 is very different than spin 1, especially at high energies!

• Quantum mechanics (or QFT) vs general relativity. Long standing clash. Write $G = 1/M_{pl}^2$ in $\hbar = c$ units. Quantum effects $\sim (GE^2)^\ell$, blow up for $E \sim M_{pl}$ ($E_{pl} = (\hbar c^5/G)^{1/2} = 1.22 \times 10^{19} GeV$) . Also many conceptual problems; black holes, meaning of quantum ideas when the metric itself can have quantum fluctuations.

String theory is the only known theory for resolving this clash, i.e. which gives a "UV completion" of quantum gravity. In string theory, replace point particles with tiny ($\ell \sim \ell_p = (G\hbar/c^3)^{1/2} = 1.62 \times 10^{-33} cm$) bits or loops of string. Turns out to lead to some bizarre consequences, like extra dimensions. Is it right? We don't know. At the very least, it is the only known well-defined theoretical framework which can be used to explore the mysteries of quantum gravity. Lessons learnt should be useful even if string theory isn't the last word on the subject. Has led to many interesting spin-offs and insights into topics which can be divorced from string theory, e.g. susy, gauge theories.

Curious history of string theory: originally developed to explain observed spectrum of mesons, e.g. $M^2 = (J + a)/\alpha'$. But found that open strings always give massless spin 1 objects, and closed strings always give massless spin 2 objects. Mesons aren't like that. But massless spin 1 objects could be the photon and gluons – good! And massless spin 2 object could be the graviton – even better – Michael Green (Cambridge) and John Schwarz (Caltech) recycled the slightly off theory of mesons into a theory of quantum gravity! Mesons are described instead by QCD. (Still interest in QCD effective string theory.)

• $x^\mu = (ct, x, y, z)$, $x_\mu = (-ct, x, y, z) = \eta_{\mu\nu} x^\mu$, $\eta_{\mu\nu}\eta^{\nu\lambda} = \delta_\mu^\lambda$. $ds^2 = -dx^\mu dx_\mu$. Lorentz vectors transform under boosts as $x'^\mu = \Lambda^\mu_\nu x^\nu$, e.g. $\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$. Can boost along any direction. Lorentz scalars, including in particular ds^2, are invariant, e.g. $ds^2 = ds'^2$.

• Light cone coordinates: $x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$. The bad: spoils rotational symmetry. The good: will make it easier to quantize string theory (avoids having to discuss here more advanced alternatives, which do not require going to the light cone coordinates). $-ds^2 = -2dx^+dx^- + dx_2^2 + dx_3^2 = -\widehat{\eta}_{\mu\nu}dx^\mu dx^\nu$. $a_\pm = -a^\mp$.

• $p^\mu = (E/c, p_x, p_y, p_z)$, with $p_\mu p^\mu = -m^2c^2$. p^μ transforms as a Lorentz 4-vector, $p^{\mu'} = \Lambda^{\mu'}_{\nu} p^\nu$. Proper time: $ds^2 = c^2 dt_p^2 = c^2dt^2(1-\beta^2)$. $u^\mu = cdx^\mu/ds = dx^\mu/dt_p = \gamma(c, \vec{v})$, and $u_\mu u^\mu = -c^2$. A massive point particle has $p^\mu = mu^\mu$. Massless particles, like the photon, have p^μ with $p^\mu p_\mu = 0$. $p_\mu x^\mu \equiv p \cdot x$ is Lorentz invariant. Free particle wavefunction $\psi \sim \exp(ip\cdot x/\hbar)$. Take $p^\pm = \frac{1}{\sqrt{2}}(p^0 \pm p^1) = -p_\mp$. So $i\hbar\partial_{x^+} \to -p_+ = E_{lc}/c$, i.e. $p^- = E_{lc}/c$.

• Extra (spacelike) dimensions, e.g. 2 extra dimensions: $-ds^2 = -c^2dt^2 + \sum_{i=1}^5 (dx^i)^2$. Consider one extra space dimension, taken to be a circle, $x \sim x + 2\pi R$. Now consider $(x, y) \sim (x + 2\pi R, y) \sim (x, y + 2\pi R)$; gives a torus. Orbifold, e.g. $z \sim e^{i\pi i/N}z$, gives a cone (singular at fixed point).

• Recall QM: $[x^i, p_j] = i\hbar\delta^{i_j}$. Particle in square well box of size a: $E = (n\pi/a)^2/2m$. Now particle in periodic box, $x_4 \sim x_4 + 2\pi R$. The other directions, x^μ, are given by some standard Hamiltonian, e.g. the hydrogen atom, which we'll call H_{4d}. So $H_{5d} = H_{4d} + \widehat{p}_4^2/2m$, with $\widehat{p}_4 = -i\hbar\partial_{x_4}$ in position space. The 4d energy eigenstates are then given by separation of variables to be $\psi_{E_{5d}}(\vec{x}, x_4) = \psi_{E_{4d}}(\vec{x})\frac{1}{\sqrt{2\pi R}}e^{i\ell x_4/R}$, with ℓ an integer, and $\psi_{E_{4d}}$ is an energy eigenstate of the 4d problem. So $E_{5d} = E_{4d} + \ell^2/2mR^2$. For R small, the low energy states are simply those with $\ell = 0$, and the extra dimension is unseen.