Physics 220, Homework 1, due on day of lecture 3

1. The group $O(n)$ is the set of all orthogonal (i.e. $g^T g = 1$) $n \times n$ matrices.

   (a) Verify that $O(n)$, with matrix multiplication, is indeed a group.

   (b) Repeat part (a) for $SO(n)$, where the S means unit determinant.

2. Find the multiplication table for a group with 3 elements, and prove that is it unique.

3. Find all essentially different possible multiplication tables for a group with 4 elements (which cannot be related by renaming elements).

4. Recall that the "order" of a group G, written in class as $|G|$, is the number of its elements. The "order" of an element $g \subset G$ is the *smallest* integer n such that $g^n = e$. Suppose someone tells you that they found an order 6 group with no order 6 elements and no order 3 elements. Show that they're mistaken.

5. Verify that if an order 6 group has an order 6 element, it must be Z_6. Verify that if an order 6 group has no order 6 element, it must be S_3.

6. Let $a = (1234)(5678)$ and $b = (1537)(2846)$ be two elements of the permutation group S_8. (Recall the notation from class: a takes the objects in positions 1,2,3,4 and moves the object in position 1 to position 2 etc.) Take products of these two elements until you fill out (or "generate") a complete group. Show that this group has order 8 and that it's isomorphic to the quaternion group, with the 8 elements $\{1, -1, i, -i, j, -j, k, -k\}$ and multiplication rule $i^2 = j^2 = k^2 = -1$, $ij = k$, $jk = i$, $ki = j$. (Here 1 is the identity element and the other multiplication rules are obvious things like $(-1)(-1) = 1$, $(-1)(i) = -i$ etc. From these it follows e.g. that $ji = j(jk) = (jj)k = -k$, so this is a non-Abelian group.) "Isomorphic" means that you can find a dictionary relating each group element, e.g. a maps to one of the quarternion elements, which respects the group multiplication table; so you're being asked to find such as dictionary or map.

7. Find the conjugacy classes of the above group.

8. Consider the conjugacy classes of S_5. Write out all elements of the conjugacy class with $\lambda_1 = 4$, $\lambda_2 = 1$. List the other conjugacy classes and their number of elements (you don't need to write out the elements explicitly, just compute their numbers), and verify that all 5! elements are accounted for.