Physics 220, Spring 2010, Homework 5, Due May 26

1. What are the two fundamental weights μ_1 and μ_2 of G_2? Write them as linear combinations of the roots α_1 and α_2. Now construct all the weights for the representation $\mu = \mu_1$, and for the representation $\mu = \mu_2$. What are the dimensions of each of these representations?

2. Georgi 8A: Find the simple roots and fundamental weights and Dynkin diagram for the algebra discussed in Georgi 6C of your last HW set, i.e. the simple Lie algebra formed by the ten matrices, σ_a, $\sigma_a\tau_1$, $\sigma_a\tau_3$, τ_2, where σ_a and τ_a are Pauli matrices in orthogonal spaces. Take the cartan generators to be $H_1 = \sigma_3$ and $H_2 = \sigma_3\tau_3$.

3. Georgi 8C: consider the algebra corresponding to the diagram for three roots α_1, α_2, and α_3, where $\alpha_1^2 = 1$ and $\alpha_3^2 = 2$, and α_2 is connected to α_1 by one line, and to α_3 by two lines. Find the Cartan matrix and the Dynkin coefficients $(2\alpha_i \cdot \mu/\alpha_i^2)$ of all of the positive roots, using the diagrammatic construction discussed in the text and in lecture. Don't forget to put the lines in the right places – this will make it harder to get confused.

4. Georgi 9A: If $|\mu\rangle$ is the state of the highest weight ($\mu = \mu_1 + \mu_2$) of the adjoint rep of $SU(3)$, show that $|A\rangle = E_{-\alpha_1}E_{-\alpha_2}|\mu\rangle$ and $|B\rangle = E_{-\alpha_2}E_{-\alpha_1}|\mu\rangle$ are linearly independent. Hint: show they're linearly independent if and only if $\langle A|A\rangle\langle B|B\rangle \neq \langle A|B\rangle\langle B|A\rangle$.

5. Find all of those regular maximal subalgebras of E_8 which can be found by omitting a single node of its extended Dynkin diagram. (Don't worry about the additional ones coming from the fact that regular maximal subalgebras can themselves have regular maximal subalgebras.)