Physics 220, Spring 2010, Homework 6, Due June 9

1. Georgi 12A. Recall that the (n, m) rep of $SU(3)$ has $\mu_{max} = n\mu_1 + m\mu_2$ and is represented by a Young Tableaux with m columns having 2 rows of boxes and n columns with 1 box row. Using the Young tableaux multiplication rule, find $(2, 1) \otimes (2, 1)$. Can you determine which reps appear antisymmetrically in the tensor product and which appear symmetrically? Also, write down the dimensions of the irreps on both sides of the multiplication.

2. Georgi 13.E. Find $[2] \otimes [1, 1]$ in $SU(N)$ and use the factors over hooks rule to check that the dimensions work out for arbitrary N.

3. Georgi 13D. Under $SU(N+M) \to SU(N)\times SU(M)\times U(1)$, we have $[1] \to ([1], [0])_M + ([0], [1])_{-N}$. The notation is $([\ell_1, \ldots], [\ell'_1, \ldots])$, where ℓ_i gives the number of boxes in the columns of the $SU(N)$ rep, and likewise for ℓ'_i with $SU(M)$. How do the other fundamental representations $[\ell]$ of $SU(N + M)$ transform under $SU(N) \times SU(M) \times U(1)$?

4. Georgi 16B. Suppose that a "quix", Q, a particle transforming like a 6 under $SU(3)_C$ exists. What kinds of bound states would you expect with this having one Q and additional quarks q or antiquarks $\bar{q}$? How do these states transform under $SU(3)_F$? Hints: the Q is a singlet under $SU(3)_F$. Also, only include states that cannot be factored into smaller color singlets, e.g. don't include $\bar{q}qqqq$, because it can fall apart into a meson $\bar{q}q$ and a baryon qqq.

5. Georgi 18.B. Under $SU(5) \to SU(3)\times SU(2)\times U(1)$, we have $5 \to (3, 1)_{-1/3} \oplus (1, 2)_{1/2}$. Find the symmetric tensor product of this with itself.

6. Similar to Georgi 27.C. You can find the anomaly $A(R)$ of the [1] representation of $SU(n)$ by calculating the anomaly of the $SU(3) \subset SU(n)$ under which the n transforms like a 3 and $n - 3$ singlets. Use this to compute the ratio of the anomaly of the N to that of the $N(N - 1)/2$ representations. Verify that the 10 and 5 have the same anomaly in $SU(5)$.

7. The result of the previous question is consistent with the fact that the anomaly vanishes for the 16 of $SO(10)$. Indeed, the anomaly must vanish for any $SO(n > 6)$ representation, because there is no d^{abc} symbol. In particular, any $SO(2n)$ spinor must have vanishing total anomaly when decomposed into irreps of the $SU(n)$ subgroup. Verify that this is the case for each of the $SO(8)$ spinors, when decomposed into $SU(4)$ representations.