Physics 220, Lecture 1

• Introduction to groups; definition of a group. Symmetry transformations, e.g. $V(x) = V(-x)$ and $G = Z_2$. Applications in physics. Quantum mechanics, atomic and molecular physics, crystals and lattices, high energy theory.

Examples of symmetries: translations, rotations, Lorentz transformations; forces.

Examples of groups: integers under addition; non-zero rationals under multiplication; rotations R_θ in a plane; rotations by $2\pi/N$ in a plane; 3d rotations.

• Abelian vs non-Abelian.

• Examples of finite groups Z_N and S_N.

Multiplication tables for Z_3 and S_3. General properties (e.g. each elements appears once and only once in every row or column).

Example of D_n, e.g. D_2 multiplication table. General D_n has $2n$ elements, generated by a and b, with $a^n = b^2 = (ab)^2 = e$. D_n can be regarded as a subgroup of $O(2)$, or as a subgroup of $SO(3)$.

• Isomorphism: map $g \to g'$ such that $g_1' g_2' = (g_1 g_2)'$.

• Subgroups $H \subset G$. Examples of Z_2 and Z_3 subgroups of S_3.

• Cayley's theorem: every G with $|G| = n$ is isomorphic to a subgroup of S_n. Proof: order group elements, then multiplication table acts on them as a permutation. E.g. D_2 is isomorphic to S_4 subgroup with elements e, $(12)(34)$, $(13)(24)$, $(14)(23)$.

• Left cosets gH of subgroup $H \subset G$, is the set of all gh for $h \in H$. Note that gH is not a group, it does not contain the identity element. Note also that it contains the same number of elements as H: $|gH| = |H|$, and none of its elements are in H. Suppose that we take a G element g' that is neither in H nor in gH, then the coset $g'H$ is also of order $|H|$, and none of its elements are in H or gH. Eventually, each and every G element fits into some coset, so we can decompose $G = H + g_1 H + \ldots g_{m-1} H$. Likewise for right cosets. This shows that $|H| = |G|/m$ with m an integer.

Examples for S_3. Take H_1 the Z_3 cyclic subset. Also consider H_2 one of the Z_2 subsets.