5/11/11 Lecture 14 outline

• Illustrate a fake, coordinate singularity: consider Minkowski space, restrict to $t > 0$. The space $t < 0$ is still there, but we'll pretend that the coordinate t doesn't cover it anymore, it just ends. But the space is fine, we just need to continue past where our bad coordinate ends. Can make it more obscure, but still the same story, by taking $t \to 1/t$ now, get $ds^2 = -dt^2/t^4 + dx^2$.

Another example that you've seen in a HW is Rindler space, $ds^2 = -x^2 dt^2 + dx^2$. Geodesics end in a finite proper time at $x = 0$. Is that a bad point? No: by a coordinate transformation $ds^2 = -dT^2 + dX^2$, it's just Minkowski space, and the original spacetime is in the wedge $|X| > T$. Just need to continue to all X and T, and no problem.

• Connect with last time, Schwarzschild black hole in Eddington Finkelstein coordinates, $t = v - r - 2M \log|\frac{r}{2M} - 1|$, where we use $G = 1$ units. Then the metric becomes

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

Schwarzschild time t ends at the horizon, but the coordinate v keeps on going, just like the proper time of an infalling observer. (Proper time is coordinate independent.)

The horizon, $r = 2GM$, $v =$ constant, is a null surface.

• Let $\widetilde{t} \equiv v - r$ and plot what happens for a collapsing star in the $(r, \widetilde{t})$ plane.

An outside observer sees the collapse only asymptotically, and the escaping light becomes increasingly redshifted. Consider light emitted at radius r_E, and received at a distant radius r_R and at time t_R, get $-4M\log(\frac{r_E}{2M} - 1) \approx t_R - r_R$. Using $\omega_R = -(u_{obs})_\mu k^\mu_\gamma$ and $\omega_E = -(u_{star})_\mu k^\mu_\gamma$, get $\omega_R \sim \omega_E e^{-t_R/4M}$.

• Matter inside $r = 2GM$ necessarily hits $r = 0$, and it happens in finite proper time,

$$\Delta\tau = \frac{1}{\sqrt{2GM}} \frac{2}{3} r^{3/2}|_0^{2GM} = \frac{2}{3}(2GM).$$

• Kruskal coordinates.

• Penrose diagram of Schwarzschild geometry.

• Penrose diagram of collapsing star.

• The geodesic equation of a test object

$$\frac{d^2 u^\alpha}{d\tau^2} + \Gamma^\alpha_{\beta\gamma} u^\beta u^\gamma = 0$$

has a counterpart for the object's spin $s^\alpha = (0, \vec{s})$,

$$\frac{ds^\alpha}{d\tau} + \Gamma^\alpha_{\beta\gamma} s^\beta u^\gamma = 0,$$

which preserves $s \cdot u = 0$.

Consider a gyroscope orbiting a star described by $ds^2_{Schwars}$. After an orbit, the spin comes back rotated by $\Delta\phi_{geodetic} = 2\pi(1 - (1 - \frac{3M}{R})^{1/2}$ per orbit.

• Spinning stars or black holes. The angular momentum J of a rotating star alters the spacetime metric. To leading order for slow rotation,

$$ds^2 = ds^2_{Schwars} - \frac{4GJ}{c^3 r^2} \sin^2\theta (rd\phi)(cdt) + \mathcal{O}(J^2).$$

The extra term leads to frame dragging: gyroscopes have additional (Lense Thirring) precession, $\vec{\Omega}_{LT} = \frac{G}{c^2 r^3}(3(\vec{J} \cdot \vec{e}_r)\vec{e}_r - \vec{J})$. Gravity probe B.

• No hair conjecture: black holes are bald, devoid of features. The only decorations they can have is gauge charges, like electric charge or angular momentum.

• Charged, spinning black hole geometry, with angular momentum J and charge Q

$$\begin{aligned} ds^2 = & -(\frac{\Delta - a^2 \sin^2\theta}{\rho^2})dt^2 - \frac{2a\sin^2\theta(r^2 + a^2 - \Delta)}{\rho^2} d\phi dt + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 \\ & + \left[\frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2\theta}{\rho^2}\right] \sin^2\theta d\phi^2, \end{aligned}$$

$$A = \frac{-Qr}{\rho^2}((dt) - a^2 \sin^2\theta d\phi).$$

where $A = A_\mu dx^\mu$ is the vector potential of E& M, $a \equiv J/M$, $\rho^2 \equiv r^2 + a^2\cos^2\theta$, $\Delta \equiv r^2 - 2Mr + a^2 + Q^2$. There are horizons where $\Delta = 0$, and there is a singularity where $\rho = 0$. If $Q^2 + a^2 > M^2$, then Δ never vanishes, so there is no horizon to cloak the singularity at $\rho = 0$ – it's a naked singularity – this would violate the cosmic censorship conjecture. But it's possible to show that such naked black holes can't form: all black holes actually have $Q^2 + a^2 \leq M^2$, so there is a horizon to cover the singularity. Attempts to through in stuff to violate $Q^2 + a^2 \leq M^2$ can be shown to fail. Cosmic censorship works in every case, even though there isn't a general proof of the conjecture. Taking $Q^2 + a^2 \leq M^2$, Δ vanishes at $r_\pm = M \pm \sqrt{M^2 - a^2 - Q^2}$, and r_+ is the true horizon. Taking $Q = 0$ for simplicity, g_{tt} vanishes on ergosphere, $r = M + \sqrt{M^2 - a^2\cos^2\theta}$; no stationary observers with timelike u^μ possible inside this radius, need to rotate with the BH.

• Black hole thermodynamics: $S_H = k_B A/4\hbar\ell_P^2$, where $\ell_P = (G\hbar/c^2)^{1/2} \approx 10^{-33}cm$. Then $dM = TdS$+work terms, with T the Hawking temperature, $k_B T_H = c^3\hbar/8\pi GM$.

Hawking showed that black holes actually aren't black: thanks to quantum effects their horizon glows as a blackbody with this temperature. Associated puzzles! To give an idea of the temperature, plug in the numbers:

$$T \approx 6 \times 10^{-8} K \left(\frac{M_{sun}}{M_{BH}} \right).$$