4/9/12 Lecture outline

⋆ Reading: Zwiebach chapters 2 and 3.

• Light cone coordinates: $x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$. The bad: spoils rotational symmetry. The good: will make it easier to quantize string theory (avoids having to discuss here more advanced alternatives, which do not require going to the light cone coordinates). $-ds^2 = -2dx^+dx^- + dx_2^2 + dx_3^2 = -\widehat{\eta}_{\mu\nu}dx^\mu dx^\nu$. $a_\pm = -a^\mp$.

• $p^\mu = (E/c, p_x, p_y, p_z)$, with $p_\mu p^\mu = -m^2c^2$. p^μ transforms as a Lorentz 4-vector, $p^{\mu'} = \Lambda^{\mu'}_\nu p^\nu$. Proper time: $ds^2 = c^2 dt_p^2 = c^2 dt^2(1-\beta^2)$. $u^\mu = cdx^\mu/ds = dx^\mu/dt_p = \gamma(c, \vec{v})$, and $u_\mu u^\mu = -c^2$. A massive point particle has $p^\mu = mu^\mu$. Massless particles, like the photon, have p^μ with $p^\mu p_\mu = 0$. $p_\mu x^\mu \equiv p \cdot x$ is Lorentz invariant. Free particle wavefunction $\psi \sim \exp(ip\cdot x/\hbar)$. Take $p^\pm = \frac{1}{\sqrt{2}}(p^0 \pm p^1) = -p_\mp$. So $i\hbar\partial_{x^+} \to -p_+ = E_{lc}/c$, i.e. $p^- = E_{lc}/c$.

• Extra (spacelike) dimensions, e.g. 2 extra dimensions: $-ds^2 = -c^2dt^2 + \sum_{i=1}^5 (dx^i)^2$. Consider one extra space dimension, taken to be a circle, $x \sim x + 2\pi R$. Now consider $(x, y) \sim (x + 2\pi R, y) \sim (x, y + 2\pi R)$; gives a torus. Orbifold, e.g. $z \sim e^{i\pi i/N} z$, gives a cone (singular at fixed point).

• Recall QM: $[x^i, p_j] = i\hbar\delta^{i_j}$. Particle in square well box of size a: $E = (n\pi/a)^2/2m$. Now particle in periodic box, $x_4 \sim x_4 + 2\pi R$. The other directions, x^μ, are given by some standard Hamiltonian, e.g. the hydrogen atom, which we'll call H_{4d}. So $H_{5d} = H_{4d} + \widehat{p}_4^2/2m$, with $\widehat{p}_4 = -i\hbar\partial_{x_4}$ in position space. The 4d energy eigenstates are then given by separation of variables to be $\psi_{E_{5d}}(\vec{x}, x_4) = \psi_{E_{4d}}(\vec{x})\frac{1}{\sqrt{2\pi R}}e^{i\ell x_4/R}$, with ℓ an integer, and $\psi_{E_{4d}}$ is an energy eigenstate of the 4d problem. So $E_{5d} = E_{4d} + \ell^2/2mR^2$. For R small, the low energy states are simply those with $\ell = 0$, and the extra dimension is unseen.