5/10/13 Lecture outline

⋆ Reading: Zwiebach chapter 7,8.

• Last time: use special choice of τ and σ such that the relativistic string EOM + constraint are

$$(\partial_\tau^2 - c^2\partial_\sigma^2)X^\mu = 0, \qquad (\dot{X} \pm cX')^2 = 0. \tag{1}$$

• Solution of the EOM for open string with free BCs at each end: imposing first at $\sigma = 0$ gives $\vec{X}(t,\sigma) = \frac{1}{2}(\vec{F}(ct+\sigma)+\vec{F}(ct-\sigma))$ where the open string has $\sigma \in [0,\sigma_1]$ and the constraint implies that $|\frac{d\vec{F}(u)}{du}|^2 = 1$, and $\vec{X}'|_{ends} = 0$ implies $\vec{F}(u+2\sigma_1) = \vec{F}(u)+2\sigma_1\vec{v}_0/c$. Note $\vec{F}(u)$ is the position of the $\sigma = 0$ end at time u/c. Then show that $\vec{v}_0$ is the average velocity of any point σ on the string over time interval $2\sigma_1/c$. Observing motion of $\sigma = 0$ end over that Δt, together with E, gives motion of string for all t. Example from book: $\vec{X}(t,\sigma = 0) = \frac{\ell}{2}(\cos\omega t, \sin\omega t)$. Find $\vec{F}(u) = \frac{\sigma_1}{\pi}(\cos\pi u/\sigma_1, \sin\pi u/\sigma_1)$, with $\vec{v}_0 = 0$. $|\frac{d\vec{F}}{du}|^2 = 1$ gives $\ell = 2c/\omega = 2E/\pi T_0$. Finally, $\vec{X}(t,\sigma) = \frac{\sigma_1}{\pi}\cos(\pi\sigma/\sigma_1)(\cos(\pi ct/\sigma_1), \sin(\pi ct/\sigma_1))$.

• Next topic: symmetries and conservation laws. Recall charge conservation $\partial_\mu j^\mu = 0$, which is related to gauge invariance, $\delta\mathcal{L} = 0$ under $\delta A_\mu = \partial_\mu f$. Recall Noether's theorem for $L(q,\dot{q})$, if continuous symmetry δq_i then $p_i\delta q_i$ is conserved. For $S = \int d\xi^0 \dots d\xi^k \mathcal{L}(\phi^a, \partial_\alpha\phi^a)$, a symmetry $\delta\phi^a$ implies a conserved current $j^\alpha = \frac{\partial\mathcal{L}}{\partial(\partial_\alpha\phi^a)}\delta\phi^a$.