5/13/13 Lecture 13 outline (that's three 13's... a lucky omen!)

⋆ Reading: Zwiebach chapter 7,8.

• Last time: symmetries and conservation laws. Recall charge conservation $\partial_\mu j^\mu = 0$, which is related to gauge invariance, $\delta\mathcal{L} = 0$ under $\delta A_\mu = \partial_\mu f$. Recall Noether's theorem for $L(q, \dot{q})$, if continuous symmetry δq_i then $p_i \delta q_i$ is conserved. For $S = \int d\xi^0 \dots d\xi^k \mathcal{L}(\phi^a, \partial_\alpha \phi^a)$, a symmetry $\delta\phi^a$ implies a conserved current $j^\alpha = \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi^a)} \delta\phi^a$.

For $\mathcal{L}_{NG}$, get conservation of $j^a_\mu = \mathcal{P}^a_\mu$ (where $a = \sigma, \tau$) is the conserved Noether current for spacetime translation invariance, $\delta X^\mu = \epsilon^\mu$. The string equations of motion are equivalent to the worldsheet conservation of this current: $\partial_a j^a_\mu = 0$. The spacetime momentum of the string is the corresponding conserved charge: $p^\mu = \int d\sigma \mathcal{P}^\tau_\sigma$. (More generally, it is $\int (\mathcal{P}^\tau_\mu d\sigma - \mathcal{P}^\sigma_\mu d\tau)$.) This is conserved for the closed string or open Neumann BCs. Not conserved for Dirichlet BCs.

The Lorentz symmetry comes from the worldsheet symmetry $\delta X^\mu = \epsilon^{\mu\nu} X_\nu$, which is a symmetry if $\epsilon^{\mu\nu} = \epsilon^{[\mu\nu]}$. The associated conserved currents are $\mathcal{M}^\alpha_{\mu\nu} = X_\mu \mathcal{P}^\alpha_\nu - (\mu \leftrightarrow \nu)$. The corresponding charges $M_{\mu\nu} = \int (\mathcal{M}^\tau_{\mu\nu} d\sigma - \mathcal{M}^\sigma_{\mu\nu} d\tau)$ are the angular momenta (and M^{0i} is related to the center of mass position at $t = 0$).

• $T_0 \equiv 1/2\pi\alpha'\hbar c$. Consider string in 12 plane. Find that the rotational angular momentum has $M_{12} = \int_0^{\sigma_1} d\sigma (X_1 \mathcal{P}^\tau_2 - X_2 \mathcal{P}^\tau_1)$, which using above $\vec{X}(t, \sigma)$ and $\vec{\mathcal{P}}^\tau = \frac{T_0}{c^2} \partial_t \vec{X}$, leads to $M_{12} = \sigma_1^2 T_0 / 2\pi c$. Since $\sigma_1 = E/T_0$ and $M_{12} = J$, this gives $J = \alpha' \hbar E^2$, which is the Regge trajectory observation of the early '70s. $\ell_s = \hbar c \sqrt{\alpha'}$.

• Aside, for later: the string worldsheet analog of $S_{particle} \supset \int q A_\mu dx^\mu$ is $S_{string} \supset -\int_\Sigma B_{\mu\nu} \partial_\tau X^\mu \partial_\sigma X^\nu d\sigma d\tau$.

• Next topic: light cone: $a^\pm = (a^0 \pm a^1)/\sqrt{2}$, so $a \cdot b = -a^- a^+ - a^+ a^- + \sum_I a^I b^I$, where $I = 2, \dots$ runs over the transverse space directions. Ugly, but can help quantize.