4/15/13 Lecture outline

⋆ Reading: Zwiebach chapters 2 and 3

• Last time: Maxwell's equations in relativistic form are $\partial_{[\mu}F_{\rho\sigma]} = 0$, and $\partial_\lambda F^{\mu\lambda} = \frac{1}{c}j^\mu$ (this convention, with indices not next to each other contracted, is peculiar to the $(-+++)$ choice of $\eta_{\mu\nu}$), which exhibits that they transform covariantly under Lorentz transformations. The scalar and vector potential combine to the 4-vector $A^\mu = (\phi, \vec{A})$ and the first two Maxwell equations are solved via $F^{\mu\nu} = \partial^{[\mu}A^{\nu]}$. The gauge invariance is $A^\mu \to A^\mu + \partial^\mu f$. E.g. Lorentz gauge: $\partial_\mu A^\mu = 0$. Physics is independent of choice of gauge, but some are sometimes more convenient than others along the way, depending on what's being done. In Lorentz gauge, the remaining Maxwell equations are $\partial_\mu \partial^\mu A^\nu = -\frac{1}{c}j^\nu$ (still some gauge freedom). In empty space we set $j^\mu = 0$ and the plane wave solutions are $A^\mu = \epsilon^\mu(p)e^{ip\cdot x}$, where $p^2 = 0$ (massless) and $p \cdot \epsilon = 0$. Can still shift $\epsilon^\mu \to \epsilon^\mu + \alpha p^\mu$, so 2 independent photon polarizations ϵ^μ.

• In QM, gauge transformation $A^\mu \to A^\mu + \partial^\mu f$ accompanies giving an overall, local phase to the QM wavefunction $\psi \to e^{iqf/\hbar c}\psi$, where q is the electric charge of the field.

• Maxwell theory and gravity in general D spacetime dimensions. $ds^2 = -c^2dt^2 + dx_1^2 + \ldots dx_{D-1}^2$. For any D, we have the same Maxwell's equations, so $F^{\mu\nu} = \partial^{[\mu}A^{\nu]}$ and $\partial_\mu F^{\mu\nu} = \frac{1}{c}j^\nu$. A point charge q has $\rho = q\delta^{D-1}(\vec{x})$ and makes an electric field with $\nabla \cdot \vec{E} = q\delta^d(\vec{x})$ in a world with $D = d+1$ spacetime dimension (the $+1$ is the time dimension, and there are d spatial directions), so $\int_{S^{d-1}} \vec{E} \cdot d\vec{a} = q$. Thus $\vec{E} = E(r)\hat{r}$ with $E(r) = q/r^{d-1}vol(S^{d-1})$, where $vol(S^{d-1}) = 2\pi^{d/2}/\Gamma(d/2)$ is the volume of a unit sphere surrounding the charge. Finally, we get that a point charge makes electric field given by $E(r) = \Gamma(d/2)q/2\pi^{d/2}r^{d-1}$. For $d = 3$, get $E(r) = q/4\pi r^2$, good.

• What about gravity in other D? In 4d, we have gravitational potential given by $V_g^{(4)} = -GM/r$, which solves $\nabla^2 V_g^{(D)} = 4\pi G^{(D)}\rho_m$. This is the gravitational potential equation in any spacetime dimension, with gravitational force taken to be $F = -m\nabla V_g$. In $\hbar = c = 1$ units, get $G = \ell_P^{D-2}$ in D spacetime dimensions. Get $G^D = GV_C$, where V_C is the compactification volume.