5/15/17 Lecture 12 outline

• Introduction to group theory: $g \in G$ with an associative multiplication rule, $g_1 \cdot g_2 = g_3$, identity element, inverse for every element. If the multiplication rule is commutative, the group is called abelian, otherwise it is called non-abelian. Lie groups have continuously many elements and are conveniently written as $g(\phi) = e^{i\phi_a T_a}$, where $T_a$ are called the generators and $\phi_a$ labels the group element. For non-Abelian Lie groups, $[T_a, T_b] \neq 0$, and since $g(\phi_a)g(\phi_b)g(-\phi_a)g(-\phi_b) \in G$, it must be the case that $[T_a, T_b] = if_{abc}T^c$ for some constants $f_{abc}$ that are called the group's structure constants. The associative property of multiplication implies that the $T_a$ need to satisfy the Jacobi identity. The simplest non-Abelian Lie group is $SU(2)$, where the $T_a$ are Hermitian, traceless, and satisfy $[T_a, T_b] = i\epsilon_{abc}T_c$. Sounds familiar, right? Yes, you saw all this in a QM class, with $T_a = L_a/\hbar$. The $|j, m\rangle$ there are an example of what is generally called a representation of the group: which is a specific realization of the multiplication rules as matrices and matrix multiplication. Symmetries necessarily correspond to groups, and that is the reason why it comes up in physics. Crystals etc use properties of discrete or point groups. Particle physicists use Lie groups. Groups will play a starring role for the rest of this class.

• More on $SU(2)$ and its representations.

• Overview of some groups, $SU(N)$, $SO(N)$.

• Continue with isospin. History: nuclear physics has a symmetry that rotates proton into neutron. Sounds weird (they have different electric charges), but the point is that the strong force, i.e. $SU(3)_C$ doesn't distinguish between $p$s and $n$s, though the electroweak force does. $SU(2)_I$ representations, for $p$ and $n$.

• The pions, in the $I = 1$ of $SU(2)_I$.

• Decomposing tensor products of $SU(2)_I$ representations; it's the same math as in addition of angular momentum in QM.

• Examples, combining protons and neutrons. Examples based on $N + N \to D + \pi$ where $D$ here denotes the deuteron.