5/17/17 Lecture 13 outline

•Recap with $N+N \to D+\pi$ where D here denotes the deuteron. Now $N+\pi \to N+\pi$.

• Quarks. $SU(2)_I$ for u and d with u and d quarks forming the $I = 1/2$ representation. Also $SU(2)_{spin}$. Find $SU(2)_I$ representations of mesons and baryons. Mesons: $\mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{3}$, and pions. Baryons: $\mathbf{2} \times \mathbf{2} \times \mathbf{2} = \mathbf{2} + \mathbf{2} + \mathbf{4}_S$. Now $SU(2)_{spin} \times SU(2)_{isospin}$.

• Including the strange quark, $SU(2)_I$ enhances to an approximate $SU(3)_F$ symmetry. Gell-Mann matrices. Isospin and hypercharge assignments of quarks and anti-quarks. Now form light mesons.

• $SU(3)$ representations.

• Approximate formula for meson masses:

$$m(q_1 q_2) \approx m_1 + m_2 + \frac{A}{m_1 m_2} \langle \vec{S}_1 \cdot \vec{S}_2 \rangle.$$

$m_u \approx m_d \approx 0.307 GeV$, $m_s \approx 0.4900 GeV$, $A \approx 0.06 GeV^3$. Note $m_{\eta', naive} \approx 355 MeV$ vs $m_{\eta', acutal} \approx 958 MeV$.

• $j = 0$ baryons and symmetry.

• Approximate formula for baryon masses:

$$m(q_1 q_2 q_3) \approx m_1 + m_2 + m_3 + A' \left(\frac{\langle \vec{S}_1 \cdot \vec{S}_2 \rangle}{m_1 m_2} + 2 - perms \right).$$

$m_u \approx m_d \approx 0.365 GeV$, $m_s \approx 0.540 GeV$, $A' \approx 0.026 GeV^3$. Comments.

• Aside on magnetic moments and magnetic dipole-dipole interactions. Recall why a classical current loop has $\vec{\mu} \propto \vec{L}$: a charge q, of mass m, moving in a circle of radius r with angular frequency ω has $\vec{L} = m\omega r^2 \hat{n}$ and $\vec{\mu} = I\pi r^2 \hat{n}$, with current $I = q/T = q\omega/2\pi$. So $\vec{\mu}_{classical} = q\vec{L}/2m$. A quantum spin has $\vec{\mu}_{quantum} = qg\vec{S}/2m$, where g is 2 for a free Dirac Fermion and quantum corrections from the interactions modify that further, e.g. for QED $g = (1 + \alpha/2\pi + \ldots)$.

• quark model predictions for magnetic moments: $\mu_p \approx \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$, $\mu_n \approx \frac{4}{3}\mu_d - \frac{1}{3}\mu_u$, and $\mu_u \approx -2\mu_d$.