5/22/17 Lecture 14 outline / summary

• Approximate formula for meson masses:

$$m(q_1 q_2) \approx m_1 + m_2 + \frac{A}{m_1 m_2}\langle \vec{S}_1 \cdot \vec{S}_2 \rangle.$$

$m_u \approx m_d \approx 0.307 GeV$, $m_s \approx 0.4900 GeV$, $A \approx 0.06 GeV^3$. Note $m_{\eta', naive} \approx 355 MeV$ vs $m_{\eta', acutal} \approx 958 MeV$.

• $j = 0$ baryons and symmetry.

• Approximate formula for baryon masses:

$$m(q_1 q_2 q_3) \approx m_1 + m_2 + m_3 + A' \left(\frac{\langle \vec{S}_1 \cdot \vec{S}_2 \rangle}{m_1 m_2} + 2 - perms \right).$$

$m_u \approx m_d \approx 0.365 GeV$, $m_s \approx 0.540 GeV$, $A' \approx 0.026 GeV^3$. Comments.

• Aside on magnetic moments and magnetic dipole-dipole interactions. Recall why a classical current loop has $\vec{\mu} \propto \vec{L}$: a charge q, of mass m, moving in a circle of radius r with angular frequency ω has $\vec{L} = m\omega r^2 \hat{n}$ and $\vec{\mu} = I\pi r^2 \hat{n}$, with current $I = q/T = q\omega/2\pi$. So $\vec{\mu}_{classical} = q\vec{L}/2m$. A quantum spin has $\vec{\mu}_{quantum} = qg\vec{S}/2m$, where g is 2 for a free Dirac Fermion and quantum corrections from the interactions modify that further, e.g. for QED $g = (1 + \alpha/2\pi + \ldots)$.

• quark model predictions for magnetic moments: $\mu_p \approx \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$, $\mu_n \approx \frac{4}{3}\mu_d - \frac{1}{3}\mu_u$, and $\mu_u \approx -2\mu_d$.

• Help with HW questions. Recall Clebsch Gordon coefficients, which can be found via $T_\pm |I, I_3\rangle = \sqrt{I(I+1) - I_3(I_3 \mp 1)} |I, I_3 \pm 1\rangle$. E.g. $|3/2, 3/2\rangle = |1,1\rangle |1/2, 1/2\rangle$. Lower both sides to get $|3/2, 1/2\rangle = \sqrt{1/3} |3/2, 1\rangle |1/2, -1/2\rangle + \sqrt{2/3} |1, 0\rangle |1/2, 1/2\rangle$, etc. compare with table of CG coefficients from the PDG. Physical observables involve squaring amplitudes, which will lead to ratios that differ by squares of the CG coefficients.

• Dirac equation for N Fermions. Explain the $U(N)$ symmetry if all have the same mass (and electric charge). (For $m = 0$, it is actually a $U(N)_L \times U(N)_R$ symmetry, at least classically. More on this later.) $SU(3)$ flavor rotates the (u, d, s) quarks. They have different charges and masses, but as far as the strong force is concerned they are all the same. This is why $SU(3)_F$ is a pretty good, but *approximate*, global symmetry.

• A few generalities about $SU(N)$ and the fundamental, anti-fundamental, and adjoint representations.

• $SU(3)$. Recall $|SU(N)| = N^2 - 1$, so $|SU(3)| = 8$. The fundamental representation is the $\mathbf{3}$ and the anti-fundamental rep is the $\overline{\mathbf{3}}$. These are the analogs of spin 1/2 for $SU(2)$;

for general $SU(N)$ the $\mathbf{N}$ and $\overline{\mathbf{N}}$ differ, but for $SU(2)$ they happen to be equivalent. For general $SU(N)$ we can think of the fundamental as acting on v^c and the anti-fundamental on $\tilde{v}_c$, and $SU(N)$ preserves δ_c^d and $\epsilon_{c_1 \ldots c_N}$ and $\epsilon^{c_1 \cdots c_N}$. For $SU(2)$, we can relate $\tilde{v}_c = \epsilon_{cd} v^d$.

• The Gell-Mann matrices and the $\mathbf{3}$ vs the $\overline{\mathbf{3}}$.

• Illustrate the $\mathbf{3}$, $\overline{\mathbf{3}}$, and $\mathbf{3} \times \mathbf{3} = \overline{\mathbf{3}} + \mathbf{6}$ via their weights in the (T_3, T_8) plane.

Next time:

• Application: approximate $SU(3)_F$ global symmetry for the (u, d, s) quarks. Mesons and baryons, spectrum and numbers.