4/24/17 Lecture 7 outline

• Last time Fermi (Dirac) -ons have : $\mathcal{L}_{Dirac} = \bar{\psi}(i\not{D} - m)\psi$, with $D_\mu = \partial_\mu + iqA^\mu$. Recall $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$. In a particular basis can take

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix},$$

where each entry is a 2×2 matrix. The equations of motion are the Dirac equation. Dropping the interaction term (perturbation theory) the plane wave solutions of the EOM are $\psi = u^s(p)e^{-ipx}$ and $\psi = v^r(p)e^{ipx}$ where $(\gamma^\mu p_\mu - m)u^s(p) = 0$ and $(\gamma^\mu p_\mu + m)v^s(p) = 0$. Useful properties include

$$\bar{u}^r(p)u^s(p) = -\bar{v}^r(p)v^s(p) = 2m\delta^{rs}, \qquad \bar{u}^r v^s = \bar{v}^r u^s = 0,$$

$$\sum_{r=1}^{2} u^r(p)\bar{u}^r(p) = \gamma^\mu p_\mu + m, \qquad \sum_{r=1}^{2} v^r(p)\bar{v}^r(p) = \gamma^\mu p_\mu - m.$$

• Quantization:

$$\psi = \sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^3 2E_p} (b^r(p)u^r(p)e^{-ipx} + c^{r\dagger}(p)v^r(p)e^{ipx}),$$

with

$$\{\psi(t,\vec{x}), \Pi(t,\vec{y})\} = i\delta^3(\vec{x}-\vec{y}), \qquad \Pi_= \partial\mathcal{L}/\partial\dot{\psi} = i\psi^\dagger.$$

Get

$$\{b^r(p), b^{s\dagger}(p')\} = \{c^r(p), c^{s\dagger}(p')\} = \delta^{rs}(2\pi)^3(2E_p)\delta^3(\vec{p}-\vec{p}'),$$

with all other zero. Find that both $b^{r\dagger}(p)$ and $c^{r\dagger}$ create spin half particles of positive energy; they are anti-particles of each other, e.g. the electron and the positron, or a quark and an anti-quark.

• Start Feynman rules for quantum electrodynamics (QED).