5/1/17 Lecture 9 outline

• Continue with Feynman rule example of $e^- + \mu^- \to e^+ + \mu^-$. Write $\mathcal{M}$ for both this and for $e^- + e^+ \to \mu^- + \mu^+$. Now compute $\mathcal{M}|^2$ averaging over initial spins and summing over final spins.

• Now connect to observables,

Consider first mean lifetime for $1 \to n$ decays, $dN = -\Gamma N dt$, $\tau = \Gamma^{-1}$, or $\Gamma_{tot} = \sum_{i=1}^n \Gamma_i$ and $\tau = 1/\Gamma_{tot}$.

Differential cross section e.g. hard sphere with $b = R\cos(\theta/2)$ and $d\sigma = bdbd\phi = R^2 d\Omega/4$ so $d\sigma/d\Omega = R^2/4$ and $\sigma = \pi R^2$. Rutherford cross section: $b = (q_1 q_2/2E)\cot(\theta/2)$ and $d\sigma/d\Omega = (q_1 q_2/4E\sin^2(\theta/2))^2$. You might have seen this in phys 110 when studying motion in a central potential.

Fermi's Golden Rule (actually, it was first derived by Dirac, but Fermi used it a lot): transition rate $= (2\pi/\hbar)|\mathcal{M}|^2$ (phase space factors). Recall $d^3\vec{n} = (V/(2\pi)^2)d^3\vec{p}$. The V factors will cancel out. But we have seen that the Lorentz invariant version is $dLIPS = \prod_i d^3p_i/(2\pi)^3(2E_i)$. The $2E$ can here be given a hand-waiving (a rigorous procedure gives the same answer) justification because the states are normalized with $\psi^\dagger\psi = \bar{\psi}\gamma^0\psi = 2E$ as you saw on a HW. So get

$$\prod_i \frac{1}{2E_i} \prod_f \frac{d^3\vec{p}_f}{(2\pi)^2 2E_f} |\mathcal{M}|^2 (2\pi)^4 \delta^4(\sum p_i - \sum p_f).$$

• E.g. decay $1 \to 2 + \dots n$ in rest frame. $d\Gamma = |\mathcal{M}|^2(2\pi)^4 \frac{S}{2m_1}\delta^4(p_1 - p_2 - p_3 \dots p_n)\prod_{j=2}^n \frac{d^3p_j}{(2\pi)^3(2E_j)}$ This is $\sim$ prob / sec. Example of $\pi^0 \to \gamma + \gamma$.

• Now consider scattering cross section for $1+2 \to 3+4+\dots$. Interaction probability is $dP = dN\sigma/A = nv_{rel}\sigma dt$; cross section σ is number of interactions per unit time per target particle divided by the incident flux. Indeed, the decay is $\sim 1/\tau = \sigma v_{rel}$ so $\sigma \sim 1/v_{rel}$. The Lorentz invariant flux factor we divide by is $4E_1E_2|\vec{v}_1 - \vec{v}_2| = 4\sqrt{(p_1\cdot p_2) - m_1^2 m_2^2}$. E.g. for $2 \to 2$ scattering in CM frame get $\sqrt{(p_1\cdot p_2)^2 - m_1^2 m_2^2} = (E_1+E_2)|\vec{p}_1|$ so

$$d\sigma = \frac{|\mathcal{M}|^2 S}{4(E_1+E_2)|\vec{p}_1|}\frac{1}{(2\pi)^2}\frac{\delta(E_1+E_2-E_3-E_4)}{(2E_3)(2E_4)}p^2 dp d\Omega.$$

• Can recover the Rutherford scattering formula as an approximation, in the limit where quantum effects are negligible (tree level) and proton recoil can be neglected so the electron is non-relativistic. Find for the spin averaged amplitude in this limit $\langle|\mathcal{M}|^2\rangle \approx m_p^2 m_e^2 e^4/p^4\sin^2(\theta/2)$ and then

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\left(\frac{1}{m_p + E_1 - E_1\cos\theta}\right)^2 \langle|\mathcal{M}|^2\rangle \approx \frac{\alpha^2}{16E^2\sin^4(\theta/2)}.$$