Find $L_{-1}|p\rangle = \eta_{\mu\nu}\alpha^{\mu}_{-1}\alpha^{\nu}_{0}|p\rangle$

$= p_\mu \alpha^{\mu}_{-1}|p\rangle$,

So we can freely shift $\varepsilon_\mu \to \varepsilon_\mu + \lambda p_\mu$

in state ~~[illegible]~~ $\varepsilon_\mu \alpha^{\mu}_{-1}|p\rangle$.

The state $\varepsilon_\mu \alpha^{\mu}_{-1}|p\rangle$ is the massless

photon, $p^\mu \varepsilon_\mu = 0$ is Gauss' law,

and $\varepsilon_\mu \to \varepsilon_\mu + \lambda p_\mu$ is gauge inv.

So get physical photon with $D-2$

physical polarization compts of ε_μ. Correct

Lorentz repo ~~with gauge~~ Illustrates general prop:

Spacetime gauge inv. $\longleftrightarrow$ Shifts by null states.

Similarly for closed string, can freely

Shift $\varepsilon_{\mu\bar\nu}\alpha^{\mu}_{-1}\bar\alpha^{\bar\nu}_{-1}|p\rangle$ by

$a_\mu \bar L_{-1}\alpha^{\mu}_{-1}|p\rangle + b_{\bar\nu} L_{-1}\bar\alpha^{\bar\nu}_{-1}|p\rangle$

which is descendent but also primary with

$h = \bar h = 1$ for any a_μ, $b_{\bar\nu}$ and $p^2 = 0$

$\Rightarrow$ Can freely shift $\varepsilon_{\mu\bar{\nu}} \to \varepsilon_{\mu\bar{\nu}} + a_\mu p_{\bar{\nu}} + b_{\bar{\nu}} p_\mu$

again, this corresponds to spacetime gauge symmetry. Write $\varepsilon_{\mu\nu}$ as 3 parts

① traceless & symmetric $\varepsilon_{\mu\nu} = S_{\mu\nu}$

$\longrightarrow$ spin 2 in spacetime = graviton!

Above gauge inv = usual reparam. gauge inv.

② antisymmetric $\varepsilon_{\mu\nu} = a_{\mu\nu}$

$\longrightarrow$ $B_{\mu\nu}$ gauge field with $B_{\mu\nu} = -B_{\nu\mu}$

"two form gauge field", with gauge inv

$B_{\mu\nu} \to B_{\mu\nu} + a_\mu p_\nu - a_\nu p_\mu$ i.e. $\underset{\text{2form}}{B} \to B + d\underset{\text{1form}}{\Lambda}$

Just as A_μ couples to worldline of pt particles via $\int A_\mu dx^\mu$, $B_{\mu\nu}$ couples to string worldvolume via $\int B_{\mu\nu} dx^\mu \wedge dx^\nu$ (more soon). B's field strength is $H = dB$, i.e. $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$.

③ $\varepsilon_{\mu\nu}$ = trace part = $\eta_{\mu\nu}$ (+ p^μ terms for $\varepsilon \cdot p = 0$)

$\uparrow$

$\eta_{\mu\nu} - p_\mu \bar{p}_\nu - \bar{p}_\mu p_\nu$

with $\bar{p}\cdot\bar{p} = 0$ & $p \cdot \bar{p} = 0$

This corresponds to the dilaton scalar Φ.

The physical state cond: primary with $h = \bar{h} = 1$, Would lead to negative norm physical states if $c_X = D > 26$

eg $\left(\alpha_{-1}\cdot\alpha_{-1} + \frac{D-1}{5}\, p\cdot\alpha_{-2} + \frac{(D+4)}{10}(p\cdot\alpha_{-1})^2\right)|p\rangle$

with $+2 + \frac{p^2}{2} = 1$. Annihilated by all $L_{n>0}$ & $h = 1$ (& need similar $\bar{\alpha}$ ops for $\bar{h} = 1$, don't bother writing here)

& This state has $\langle\phi|\phi\rangle = \frac{2}{25}(D-1)(26-D)\langle p|p\rangle$

(Related to sign of $c_T \to$ Liouville kinetic term)

$\Rightarrow$ Better not have $D > 26$! (unless this state is somehow eliminated)

"No ghost thm" for $D \leq 26$

no neg norm physical sts if $D \leq 26$

For $D = 26$ many null states eg.

$|\psi\rangle = \left(L_{-2} + \frac{3}{2}L_{-1}^2\right)|\chi\rangle$ is primary & has $h = 1$ if $|\chi\rangle$ is primary with $h = -1$. $|\psi\rangle$ = Primary & descendant

$\therefore$ $\langle\psi|\psi\rangle = 0$. Null states $\to$ extra gauge symm.

For $C_X = D < 26$ Find physical states condition eliminates 1 dimension worth of oscillators $\sim D-1$ indep oscillators.

For $C_X = D = 26$, Extra gauge symm of add'l null states eliminates another dim worth of oscillators $\longrightarrow D-2=24$ eff. ~~as~~ indep. oscillators. Expected $D-2$ "light cone" ~~dim~~ dimensions since τ & σ worldsheet reparam inv can be used to eliminate 2 polarization directions.

Will give better explanation soon via ghosts.

$$C_X + C_{ghosts} + C_{Liouville} = 0$$

$$C_{ghosts} = -26 \qquad C_X = D$$

Can neglect $C_{Liouville}$ if $D=26$. Otherwise it is the dim. not eliminated by extra gauge symms.

Strings in curved spacetime with general (closed string) backgrounds for massless fields: graviton, dilaton, antisymm tensor

$$S_{ws} = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{\det h} \Big[\big(h^{ab} G_{\mu\nu}(X) + i\varepsilon^{ab} B_{\mu\nu}(X) \big) \cdot$$

$$\cdot\, \partial_a X^\mu \partial_b X^\nu + \alpha' R\, \Phi(X) \Big]$$

↑ worldsheet Ricci scalar

~~[illegible]~~ S invariant under $\delta B_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$ for any Λ_μ.

Expanding $G_{\mu\nu}(X) = \eta_{\mu\nu} + S_{\mu\nu}(X)$

the coeff of $S_{\mu\nu}(X) = S_{\mu\nu} e^{ipX}$ is our graviton vertex operator

$S_{\mu\nu} \partial X^\mu \bar{\partial} X^\nu e^{ipX} \longrightarrow S_{\mu\nu} \alpha_{-1}^\mu \tilde{\alpha}_{-1}^\nu |p\rangle$

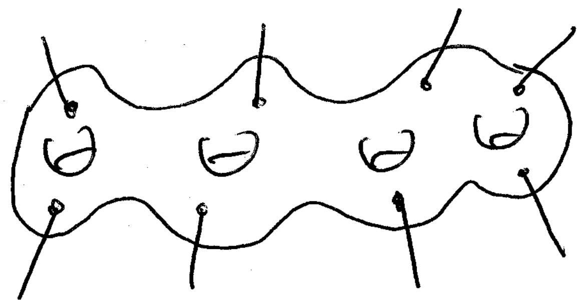
etc. for $B_{\mu\nu}$ & Φ, these are coherent backgrounds of our massless closed string states.

The dilaton plays a special role in String theory. Suppose eg $\Phi = \Phi_0$ a const.

The Φ_0 term in S_{WS} is $\Phi_0 \int \frac{d^2\sigma}{4\pi} \sqrt{\det h}\, R$

& $\int \frac{d^2\sigma}{4\pi} \sqrt{\det h}\, R$ is a topological invt. of 2d manifolds $\equiv \chi$ = "Euler character"

$\left(\frac{\delta}{\delta h_{ab}} \rightarrow R_{ab} - \frac{1}{2} h_{ab} R \equiv 0 \text{ in 2d} \right)$



Suppose there are L ~~holes~~ handles - this is the worldsheet for a L loop string scattering process. The —— are operator insertions - suppose there are n of these for a scattering process involving n fields in spacetime.

$$\chi = 2(1-L) - n$$

\# holes or "punctures"

\# of handles or "genus"

So $e^{-S_{WS}}$ weights this process by

$$e^{\Phi_0 (n + 2(L-1))}$$

Can generally show an L loop contribution to effective action $\sim g_{\text{string}}^{2(L-1)}$

Writing $e^{-\frac{1}{g_s^2} S_{S.T.}}$ every interaction $\sim 1/g_s^2$ & every propagator $\sim g_s^2$ & L loop diag has $L-1$ more propagators than vertices,

So the spacetime coupling constant of strings is $g_s = e^{\Phi_0}$. ($e^{n\Phi_0}$ term $\Rightarrow$ vertex ops have factor of g_s).

So the spacetime coupling constant of string theory is not a parameter but instead the expectation value of a field, the dilaton Φ!

" String thy. has no free parameters "

This is in principle, since all parameters are really expectation values of fields. Should be dynamically determined. E.g. Would like to see generated potential

Φ = weak coupling in this dir, so $V \to 0$.

Vacuum explains observed values of coupling constants. Better happen that Φ has a reasonably large mass in minimum since we think light scalars are a problem with 5th force non-observation. (Problem: in simplest cases to analyze, $V(\Phi)$ looks very flat...)

The terms in $S_{WS} = \int \frac{d^2\sigma}{4\pi\alpha'} \sqrt{\det h}\left(h^{ab} G_{\mu\nu} + i\varepsilon^{ab} B_{\mu\nu}\right) \cdot \partial_a X^\mu \partial_b X^\nu + \alpha' R \, \Phi(X)\Big)$ are a "2d nonlinear sigma model" ~~these~~ which generally is not scale invariant: the "couplings" $G_{\mu\nu}(X)$, $B_{\mu\nu}(X)$, and $\Phi(X)$ generally change or "flow" with the Renormalization group scale of 2d W.S. theory. This is not what we want for String theory - we want the Worldsheet action to be scale invariant. This condition gives the spacetime equations of motion: Einsteins eqns for gravity & generalizations.

Warmup example: 2d $O(N)$ model

$$S = \frac{1}{2e_0^2} \int d^2\sigma \sqrt{\det h_{ab}}\, \partial_a n^i \partial_b n_i h^{ab}$$

with $n^i n_i \equiv \vec{n}\cdot\vec{n} = 1$ constraint & n a N dim'l vector. The σ model "target space" here is a $N-1$ dim'l sphere in N dimensions. e_0^{-1} is the radius of the sphere. Large radius $\sim$ weak coupling, small radius $\sim$ strong coupling. Can compute at 1 loop that e_0 is a running coupling: $e_0 \to e(\mu)$ with $\mu = $ 2d energy scale & $\frac{de}{d\ln\mu} = -\frac{(N-2)}{4\pi} e^3$

$$\Rightarrow \quad e^{-2}(\mu) = \frac{N-2}{2\pi} \ln\left(\frac{\mu}{\Lambda}\right) \quad (\text{for } \mu > \Lambda)$$

in U.V. asymptotically free, since $\lim_{\mu\to\infty} e(\mu) \to 0$, & goes to strong coupling in IR. Find in IR: <u>mass gap</u> with $O(N)$ restored rather than broken to $O(N-1)$ by $\langle \vec{n} \rangle$.

Massive particles in N dim'l vector rep of $O(N)$. The running $e(r)$ corresponds to $T_{z\bar{z}} = -\frac{\beta(e)}{2e_0^3} h^{ab} \partial_a \vec{n} \partial_b \vec{n}$

with $\beta(e) = -\frac{(N-2)}{4\pi} e^3 \neq 0 \implies T_{z\bar{z}} \neq 0$

not scale invt. clearly.

More generally: 2d σ model with

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{\det h}\, h^{ab} G_{\mu\nu}(X) \partial_a X^\mu \partial_b X^\nu$$

has $T_{z\bar{z}} = -\frac{1}{2\alpha'} \beta^G_{\mu\nu} h^{ab} \partial_a X^\mu \partial_b X^\nu$

with $\beta^G_{\mu\nu} = -\alpha' R_{\mu\nu}$ to 1 loop in 2d σ model perturbation theory (not 1 loop in spacetime! tree level in space time.) Here $R_{\mu\nu}$ = spacetime Ricci tensor. So 2d scale invariance is broken unless the target space metric $G_{\mu\nu}(X)$ is <u>Ricci flat</u>, $R_{\mu\nu} = 0$.

Outline of $\beta^G_{\mu\nu}$ calc: Expand X^μ around classical sol'n $X^\mu = X^\mu_{cl} + \tilde{X}^\mu$

fluctuations

$$G_{\mu\nu}(X) = G_{\mu\nu}(X_{cl}) + G_{\mu\nu,\rho}(X_{cl})\tilde{X}^\rho + G_{\mu\nu\rho\sigma}\tilde{X}^\rho\tilde{X}^\sigma \cdots$$

get e.g. $G_{\mu\nu,\rho\sigma}\tilde{X}^\rho\tilde{X}^\sigma\partial^a\tilde{X}^\mu\partial_a\tilde{X}^\nu \rightarrow$ 

1 loop  renormalizes

$G_{\mu\nu}(X_{cl})\,\partial^a\tilde{X}^\mu\,\partial_a\tilde{X}^\nu$ kinetic term.

σ model loop expansion parameter: $\sqrt{\alpha'}/R_c$ where R_c is radius of curvature of spacetime metric $G_{\mu\nu}$. String thy has 2 perturbation thy expansion parameters

$\sqrt{\alpha'}/R_c$: Worldsheet

$g_s = e^{\Phi}$: spacetime $\leftarrow$ # of handles in worldsheet

Here considering tree level in g_s &

1 loop in worldsheet exp. parameter.