Week 3 Homework (Due Oct. 19)

Consider a supersymmetric quantum mechanics with two conserved complex supercharges (up to now, we've been considering only one conserved supercharge), Q_α, for $\alpha = 1,2$, such that $Q_\alpha^2 = 0$ and $\{Q_\alpha, Q_\beta^\dagger\} = 2H\delta_{\alpha\beta}$. Write the superspace in terms of supercoordinates t, θ_α, and θ_α^*. The supercharges and supercovariant derivatives are

$$Q_\alpha = \frac{\partial}{\partial\theta_\alpha} + i\theta_\alpha^*\frac{\partial}{\partial t}, \qquad Q_\alpha^\dagger = \frac{\partial}{\partial\theta_\alpha^*} + i\theta_\alpha\frac{\partial}{\partial t},$$

$$D_\alpha = \frac{\partial}{\partial\theta_\alpha} - i\theta_\alpha^*\frac{\partial}{\partial t}, \qquad D_\alpha^\dagger = \frac{\partial}{\partial\theta_\alpha^*} - i\theta_\alpha\frac{\partial}{\partial t}.$$

Define a *chiral superfield* to be a *complex* field Φ which satisfies $D_\alpha^\dagger\Phi = 0$ for $\alpha = 1,2$. Likewise, an *anti-chiral superfield* is one which satisfies $D_\alpha\overline{\Phi} = 0$. Note that $D_\alpha^\dagger(\theta_\beta) = 0$.

a) Verify that $D^\dagger T = 0$ for $T \equiv t \mp i\theta_1\theta_1^* \mp i\theta_2\theta_2^*$. We can write this as $T = t \mp i\theta_\alpha\theta_\alpha^*$, summing the repeated indices (it looks even nicer if we write $\theta^{*\alpha}$, but I won't bother).

b) Write the general chiral superfield as $\Phi(\theta_\alpha, T) = \phi(T) + \theta_\alpha\psi_\alpha(T) + \theta_1\theta_2F(T)$. Expand this out to write it in terms of the fermionic coordinates and the functions $\phi(t)$, $\psi_\alpha(t)$, and $F(t)$ and their derivatives.

c) Verify that $\overline{\Phi} \equiv \Phi^\dagger$ is an anti-chiral field.

d) Verify that $\int dt d\theta_1 d\theta_2 W(\Phi, \Phi^\dagger)$ respects the Q_α supersymmetries for any function $W(\Phi, \Phi^\dagger)$. Verify that it respects the $Q_\alpha^\dagger$ supersymmetries if and only if $\partial W/\partial\Phi^\dagger = 0$, i.e. $W = W(\Phi)$, independent of $\Phi^\dagger$. Likewise, verify that $\int dt d\theta_1^* d\theta_2^* \overline{W}$ respects all supersymmetries if and only if W is a function only of the anti-chiral field $\overline{\Phi} = \Phi^\dagger$. Finally, verify that $\int dt d^2\theta d^2\theta^* K(\Phi, \overline{\Phi})$ respects supersymmetry for any function K, without restrictions. Hint: you can save ink by using the above superderivative expressions for the action of the supersymmetry generators and noting that $\int dt dG/dt = 0$ and $\int d\theta dG/d\theta = 0$ for any superfunction G.

Consider the action $S = \int dt\mathcal{L}$, with

$$\mathcal{L} = \int d^4\theta K(\Phi, \overline{\Phi}) + \int d^2\theta W(\Phi) + \int d^2\theta^* W(\overline{\Phi}).$$

e) Do the θ_α and θ_α^* integrals to write out the action for the component fields ϕ, and ψ_α, and ψ_α^*. Eliminate the auxiliary fields F and F^* by their equations of motion.