Seiberg's argument for W nonrenormalization:

① Promote all coupling constants g_K to background expectation values of chiral superfields.

② Susy $\Rightarrow$ $W(\Phi_i, g_K)$ must be holomorphic in both chiral superfields Φ_i & coupling constants g_K since g_K are now chiral superfields

③ Impose symmetries & known limits

Find this often suffices to determine the <u>exact</u> quantum effective superpotential!

E.g. consider $W_{cl} = \frac{1}{2} m \Phi^2 + \frac{1}{3} g \Phi^3$

W_{eff} should be a holomorphic function of Φ and also m and g. Now consider global symmetries

	$U(1)_\Phi$	$U(1)_R$
Φ	1	1
m	-2	0
g	-3	-1

$\Rightarrow$ $W_{exact} = m\Phi^2 f\left(\frac{g\Phi}{m}\right)$

Limits: For $g \to 0$, should get $W_{eff} \to W_{cl}$

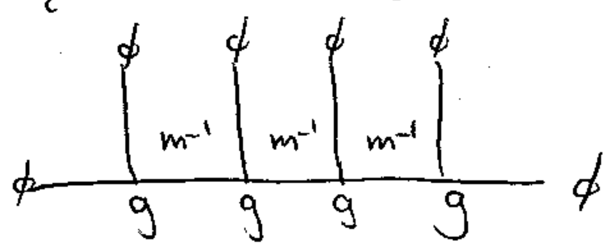
so $W_{eff} = m \Phi^{2} f\left(\frac{g\Phi}{m}\right)$

with $f(t) \to \frac{1}{2} + \frac{1}{3} t$ for $g \to 0$

but by taking $m \to 0$, can get any value for $t = g\Phi/m$ in $g \to 0$ limit. So

$f(t) = \frac{1}{2} + \frac{1}{3} t$ for all t

$\therefore \quad W_{exact} = W_{cl} = \frac{m}{2}\Phi^{2} + \frac{1}{3} g \Phi^{3}$

note diagrams such as 

not included since not 1PI. $m \to 0$ limit is smooth.

Actually W_{exact} is the "Wilsonian effective superpotential". Integrate out modes with momentum $p > \mu$.

↳ Keep μ finite & continue to $\mu = 0$

Eliminates possible IR divergences..

"Integrating out" a heavy field H for energies below its mass m

$$\int [DL_i][dH]\, e^{-S_{cl}[L_i, H]} = \int [DL_i]\, e^{-S_{eff}[L_i]}$$

$$S_{eff}(L_i) = \int d^4x\, \mathcal{L}_{eff}(L_i)$$

$$\mathcal{L}_{eff}(L_i) = \int d^4\theta\, K_{eff} + \int d^2\theta\, W_{eff} + h.c.$$

Nonrenormalization of $W \longrightarrow$

$$W_{eff}(L_i) = W(L_i, H)\Big|_{H = \langle H \rangle}$$

with $\langle H \rangle$ determined by $\frac{\partial W(L_i, H)}{\partial H} = 0$

i.e. the heavy field H just sits in its minimum. Its quantum fluctuations around the minimum affect K but not W.

E.g. $W = \frac{1}{2} m H^2 + \lambda L^2 H \longrightarrow W_{eff} = \frac{-\lambda^2}{2m} L^4$

Now consider $W_{eff}(\Phi_i)$ effective action for light fields. Is susy spontaneously broken?

Only if there is no sol'n $\langle \Phi_i \rangle$ such that

$$\frac{\partial}{\partial \Phi_i} W_{eff} \Big|_{\Phi_j = \langle \Phi_j \rangle} = 0 \text{, for all } i$$

This is n eqns for n unknowns $\langle \Phi_i \rangle$ $i = 1 \ldots n$. If W_{eff} is generic, there will be sol'ns.

E.g. $W = \lambda \Phi$ ← no susy sol'n, but not generic

$W = \sum_n a_n \Phi^n$ ← yes susy sol'n.

U(1) symmetries don't help:

$$W(\Phi_1, \ldots \Phi_n) = \hat{W}\left(\frac{\Phi_K}{(\Phi_1)^{q_K/q_1}}\right) \quad K = 2 \ldots n$$

now $\partial_i W = 0 \longrightarrow$ $n-1$ eqns for $n-1$ unknowns.

$U(1)_R$ symmetries do help.

$U(1)_R \Rightarrow W_{eff} = (\Phi_1)^{2/q_1} \hat{W}(U_2, \ldots U_n)$

$U_k \equiv \Phi_k / (\Phi_1)^{q_k/q_1}$ = neutral

now $\partial_i W_{eff} = 0 \longrightarrow$ n eqns for the n-1 unknowns $U_k \longrightarrow$ can break susy.

Extra eqn : $\dfrac{\partial W}{\partial \Phi_1} = 0 \rightarrow \hat{W} = 0.$

provided $\langle \Phi_1 \rangle \neq 0$ (to define U_k)

$\overset{!}{\varepsilon}$ $q_1 \neq 0$. This means that the $U(1)_R$ symmetry is also spontaneously broken, along with supersymmetry.

Get massless R-axion along with massless goldstino. (In SUGRA both of these get masses.)

E.g. $W = \lambda \Phi$ has broken susy

massless fermion $\psi \rightarrow$ goldstino

massless scalar $\phi \rightarrow$ R axion.

Vector superfields: combine A_μ ← gauge field

with superpartner gaugino λ_α. For massless $A_\mu \to 2$ polarizations so same # of degrees of freedom as 2 compt. fermion.

Susy QED without matter:

$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{g^2} \lambda^\dagger_{\dot\alpha} \left(-i \partial^{\dot\alpha\alpha}\right) \lambda_\alpha$$

$$F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$$

neutral photino.

free photon + free neutral photino.

Can also add $\frac{\theta}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$ θ term.

This all comes from a vector superfield V satisfying $V = \bar{V}$, and the gauge inv

$V \to V + i(\Lambda - \bar{\Lambda})$ with Λ a chiral superfield.

$$V = B + \theta\chi + \bar\theta\bar\chi + \theta^2 C - \theta\sigma^\mu\bar\theta A_\mu$$
$$+ i\theta^2\bar\theta\left(\bar\lambda + \tfrac{1}{2}\bar\sigma^\mu\partial_\mu\chi\right) - i\bar\theta^2\theta\left(\lambda - \tfrac{1}{2}\sigma^\mu\partial_\mu\bar\chi\right)$$
$$+ \tfrac{1}{2}\theta^2\bar\theta^2\left(D + \partial^2 B\right)$$

Get $\delta B = i(\Lambda - \bar{\Lambda})$
$\delta \chi = i\sqrt{2}\,\psi_\Lambda$
$\delta C = i F_\Lambda$

B, χ, C = gauge artifacts

"Wess Zumino Gauge": $B = \chi = C = 0$

$\delta A_\mu = \partial_\mu (\Lambda + \bar{\Lambda})$ ← usual gauge transf for photon

$\delta \lambda = 0$ ← photino neutral

$\delta D = 0$ ← gauge invt. aux. field

Field Strength $W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V$ is gauge invt.

$$\bar{W}_{\dot\alpha} = -\frac{1}{4} D^2 \bar{D}_{\dot\alpha} V$$

also $\bar{D}\, W_\alpha = 0$ & $D\,\bar{W}_{\dot\alpha} = 0$

Chiral & antichiral superfields. Also superspace Bianchi identity: $D^\alpha W_\alpha = \bar{D}_{\dot\alpha} \bar{W}^{\dot\alpha}$

In "Wess Zumino" gauge:

$$W_\alpha = -i\lambda_\alpha(y) + \theta_\alpha D - \frac{i}{2} F_{\alpha\beta}\theta^\beta + \theta^2 \partial_{\alpha\dot\alpha}\bar{\lambda}^{\dot\alpha}$$

Let $\tau \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$

Then $\int d^2\theta \; \frac{-i\tau}{16\pi} W_\alpha W^\alpha = -\frac{1}{4g^2} F_{\mu\nu}{}^2$

$+ \frac{\theta}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{i}{g^2} \bar{\lambda}^{\dot\alpha} \partial_{\dot\alpha\alpha} \lambda^\alpha + \frac{1}{2g^2} D^2.$

D has no kinetic terms → aux field.

$\frac{\partial \mathcal{L}}{\partial D} = 0 \longrightarrow D = 0,$ Susy unbroken

A Fayet Iliopoulos term is

$\Delta \mathcal{L} = \xi \int d^4\theta \, V$ Only allowed for Abelian gauge thys.

Adds ξD term to $\mathcal{L}$.

Now $\frac{\partial \mathcal{L}}{\partial D} = 0 \longrightarrow \frac{1}{g^2} D + \xi = 0$

$\rightarrow \langle D \rangle = -g^2 \xi \neq 0$ breaks susy since $D = Q(\text{something})$

Now add charged chiral superfields Φ_i with charges q_i.

$$\Phi_i = \phi_i(y) + \sqrt{2}\,\theta\psi_i(y) + \theta^2 F_i(y)$$

ϕ_i, ψ_i, and F_i each have charge q_i since the supercharges Q_α & $\bar{Q}_{\dot\alpha}$ are gauge invt.

Under gauge transformation $V \to V + i(\Lambda - \bar{\Lambda})$

$$\Phi_i \to e^{-iq_i\Lambda}\,\Phi_i$$
$$\bar{\Phi}_i \to e^{iq_i\bar{\Lambda}}\,\bar{\Phi}_i$$

Canonical gauge invt. Φ_i kinetic terms from replacing usual Kahler potential $K = \sum_i \bar{\Phi}_i \Phi_i$

with $K = \sum_i \bar{\Phi}_i\, e^{q_i V}\,\Phi_i$.

$$\int d^4\theta\, K = \sum_j \Big[\bar{F}_j F_j - |(\partial_\mu + iq_j A_\mu)\phi_j|^2$$
$$- i\bar{\psi}^j_{\dot\alpha}(\partial^{\dot\alpha\alpha} + iq A^{\dot\alpha\alpha})\psi^j_\alpha - \frac{i}{\sqrt{2}} q_j(\phi_j \bar{\lambda}\bar{\psi}_j - \bar{\phi}_j \lambda \psi_j)$$
$$+ \frac{1}{2} q_j D \bar{\phi}_j \phi_j \Big]$$

Integrate out D terms by D EOM

$$\langle D\rangle = -\frac{g^2}{2}\left(2\xi + \sum_i q_i |\phi^i|^2\right)$$

possible Fayet Iliopoulos term

Unbroken susy requires $\langle D\rangle = 0$

The potential has a contribution from plugging back in $\langle D\rangle$

$$V_D = + \frac{1}{2g^2}\langle D\rangle^2 = \frac{g^2}{8}\left(2\xi + \sum_i q_i |\phi_i|^2\right)^2$$

Total scalar potential if superpotential is also added:

$$V = V_F + V_D \qquad \text{with } V_F = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2$$

note

$V_F \geq 0$ & $V_D \geq 0$ So $V \geq 0$ and

$V = 0$ iff $V_F = 0$ & $V_D = 0$ iff $\langle F_i\rangle = 0$ and $\langle D\rangle = 0$

Unbroken susy conditions.