1/22/07 Lecture 4 outline

• Last time:

$$Z[J] = N \int [d\phi] e^{\frac{i}{\hbar}(S+\hbar \int d^4x J\phi)} = N \exp[\frac{i}{\hbar} S_{int}[-i\frac{\delta}{\delta J}]) Z_{free}[J],$$

where N is an irrelevant normalization factor (independent of J). Correspondingly, the green's functions are given by

$$G^{(n)}(x_1 \dots x_n) = \frac{\int [d\phi]\phi(x_1)\dots\phi(x_n)\exp(\frac{i}{\hbar}S_I[\phi])\exp[\frac{i}{\hbar}S_{free}]}{\int [d\phi]\exp(\frac{i}{\hbar}S_I[\phi])\exp[\frac{i}{\hbar}S_{free}]},$$
$$= \frac{1}{Z[J]} \prod_{j=1}^{n} \left(-i\hbar \frac{\delta}{\delta J(x_j)}\right) \cdot Z[J]\big|_{J=0}.$$

(The denominator (in both lines) cancels off the vacuum bubble diagrams, which don't depend specifically on the Green's function.)

• Illustrate the above formulae, and relation to Feynman diagrams, e.g. $G^{(1)}$, $G^{(2)}$ and $G^{(4)}$ in $\lambda\phi^4$ theory. The functional integral accounts for all the Feynman diagrammer; even symmetry factors etc. come out simply from the derivatives w.r.t. the sources, and the expanding the exponentials.

• Recall story of cancellation of bubble diagrams. Recall for computing S-matrix elements, we will especially be interested in *connected* Green's functions. There are nice combinatoric formulae (you might have already seen some last quarter?). E.g.

$$\sum \text{all diagrams} = \left(\sum \text{“connected”}\right) \cdot \exp(\sum \text{disconnected vacuum bubbles}).$$

And the vacuum bubble diagrams cancel. We write “connected” because for $n > 2$ point functions there are still disconnected diagrams, connected to the external points, included in this sum.