1/29/07 Lecture 6 outline

- Recall

$$e^{iW[J]} = N \int [d\phi] e^{\frac{i}{\hbar}\left(S[\phi] + \int J\phi\right)},$$

(here we rescaled $J$ by factor of $1/\hbar$ compared with before).

- Suppose diagram has $I$ internal lines, $V$ vertices, $L$ loops. Connected graphs have $L = I - V + 1$. Graphs go like $\hbar^{-V}\hbar^{I} = \hbar^{L-1}$. So $W[J] = W_{-1}\hbar^{-1} + W_0 + \hbar W_1 + \ldots$, where $W_{-1}$ are tree-graphs (no loops), $W_0$ gives the 1-loop graphs, etc.

- Example: free Klein Gordon theory. We found $Z[J]$ above. Then

$$W[J] = i\tfrac{1}{2}\hbar^{-1} \int d^4x \int d^4y J(X) D_F(x-y) J(y).$$

(Rescaled source $J$ compared with before.)

We see that the only connected Green's function in this case is the 2-point function:

$$G^{(2)}_{free}(x,y) \equiv G(x-y) = \hbar D_F(x-y).$$

In an interacting theory, like $\lambda\phi^4$,

$$G^{(2)}(x,y) = \hbar D_F(x-y) + O(\lambda) \text{ corrections.}$$

- Emphasize that tree graphs are classical. Example: consider $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \phi J$, with the source term $J$. The classical field EOM is

$$(\partial_\mu\partial^\mu + m^2)\phi_c = -\frac{1}{3!}\lambda\phi_c^3 + J(x).$$

We can solve this classically to zero-th order in $\lambda$ as

$$\phi_c^{(0)}(x) = \int d^4y D_F(x-y) iJ(y),$$

where $(\partial_\mu\partial^\mu + m^2)D_F(x-y) = -i\delta(x-y)$. To solve to next order in $\lambda$, we plug this back into the above:

$$\phi_c^{(1)}(x) = \phi_c^{(0)}(x) - i\frac{1}{3!}\lambda \int d^4y D_F(x-y)\phi_c^{(0)}(y)^3$$

Continue this way, this can be represented as a sum of tree-level diagrams, with one $\phi$ and different numbers of $J$'s on the external legs. This is perturbation theory for the classical field theory.

- Examples of diagrams contributing to $G^{(n)}_{conn}$ for $n = 2, 4, 6$, in $\lambda\phi^4$.

• It is useful to define a further specialization of the diagrams, those that are 1PI: one - particle irreducible. The definition is that the diagrams is connected, and moreover remains connected upon removing any one internal progagator (and amputating all external legs).

•Examples of $n = 2, 4, 6$ point 1PI diagrams in $\lambda\phi^4$.

• In momentum space, it is defined from the 1PI diagram, with all external momenta taken to be incoming:

$$\text{1PI diagram} \equiv i\tilde{\Gamma}^{(n)}(p_1, \dots p_n),$$

where the external propagators are amputated, and the $(2\pi)^4\delta^4(\sum_i p_i)$ is omitted. If there is an interaction like $g\phi^n/n!$, then, at tree-level, $\tilde{\Gamma}^{(n)} = g$. Special definition for case $n = 2$ : we define the 1PI diagram to be $-i\Pi'(p)$, and we instead define

$$i\tilde{\Gamma}^{(2)}(p, -p) = \text{1PI diagram} + i(p^2 - m^2) = i(p^2 - m^2 - \Pi'(p^2)).$$

Define position space 1PI diagrams by Fourier transform. They correspond to

$$\Gamma^{(n)}(x_1, \dots x_n) = \langle T\phi(x_1)\dots\phi(x_n)\rangle|_{1PI}.$$

• 2-point function, via summing geometric series:

$$D(p) = \frac{i}{\tilde{\Gamma}^{(2)}} = \frac{i}{p^2 - m^2 - \Pi'(p^2)}.$$

$-i\Pi'$ is computed from the 1PI diagrams. $\Pi'(p^2)$ is called the self-energy, like momentum dependent mass term. The special definition of $\tilde{\Gamma}^{(2)}$ is because $D(p) = i/\tilde{\Gamma}^{(2)}$ will be nice, and allow extending to higher point functions.

• The point of the 1PI diagrams is that the quantum loop corrections are simply obtained by replacing the vertices with the 1PI greens functions! Indeed, Draw pictures for $n = 2, 4, 6$ point functions. Obtain the full $W[J]$ via tree-graphs assembled from the 1PI building blocks.