2/20/09 Lecture 13 outline

• Last time,

$$\Pi'(p^2) = -\frac{\lambda m^2}{32\pi^2}\left(\frac{2}{\epsilon} - \log\frac{m^2}{4\pi\mu^2} + 1 - \gamma\right) + \text{more loops}$$

$$\tilde{\Gamma}^{(4)} = -\lambda + \frac{\lambda^2}{32\pi^2}\left(\frac{6}{\epsilon} - 3\gamma + 3\log\frac{4\pi\mu^2}{m^2} + A_1(s) + A_1(t) + A_1(u)\right) + \text{more loops},$$

where

$$A_1(s_E) = 2 - \sqrt{1+4m^2/s_E}\log\left(\frac{\sqrt{1+4m^2/s_E}+1}{\sqrt{1+4m^2/s_E}-1}\right).$$

• Renormalization. The input to the functional integral is the "bare" lagrangian. It is not physically observable, because we observe quantities like mass, charge, etc. with all the quantum corrections included. Write the largrangian for the bare fields as:

$$\mathcal{L}_B = \tfrac{1}{2}\partial_\mu\phi_B\partial^\mu\phi_B - \tfrac{1}{2}m_B^2\phi_B^2 - \lambda_B\frac{1}{4!}\phi_B^4.$$

The bare field is related to the physical one by $\phi_B \equiv Z_\phi^{1/2}\phi$. We can view this as

$$\mathcal{L}_B = \mathcal{L}_{phys} + \mathcal{L}_{c.t.}$$

where

$$\mathcal{L}_{phys} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}m^2\phi^2 - \lambda\frac{1}{4!}\phi^4$$

involves the physical field, mass, coupling constant. What's left are the counterterms:

$$\mathcal{L}_{c.t.} = \tfrac{1}{2}(Z_\phi - 1)\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}(m_B^2 Z_\phi - m^2)\phi^2 - (\lambda_B Z_\phi^2 - \lambda)\frac{1}{4!}\phi^4.$$

• Define $\delta_Z \equiv Z_\phi - 1$, $\delta m = m_B^2 Z_\phi - m^2$, $\delta\lambda = \lambda_B Z_\phi^2 - \lambda$. There are extra diagram contributions for these corrections. There is a line (like the propagator) with an insertion of the counterterm, which gives a factor of $i(p^2\delta_Z - \delta_m)$. There is a new vertex with a factor of $-i\delta_\lambda$. These new diagrams count as having one loop factor (one factor of $\hbar$).

• Among other things, these corrections cancel the divergences. E.g.

$$\delta_m = \frac{\lambda m^2}{16\pi^2}\frac{1}{\epsilon} + \text{finite} + \mathcal{O}(\lambda^3).$$

$$\delta_\lambda = 3\frac{\lambda^2}{16\pi^2}\frac{1}{\epsilon} + \text{finite} + \mathcal{O}(\lambda^4).$$

To one loop, $\delta_Z = 0+$(finite), because $\Pi'(p^2)$ is independent of $p^2$.

• What to do about the finite parts is a choice that we can make, our renormalization prescription. We have to define what we're calling the physical mass and coupling. One choice is to define the mass to be the pole of the full propagator (sum of all connected diagrams), $D(p) = i/\tilde{\Gamma}^{(2)}$, and to define the physical field so that the residue of the pole is $i$. This means

$$\Pi'(m^2) = 0, \qquad \frac{d\Pi'}{dp^2}|_{p^2=m^2} = 0, \qquad \tilde{\Gamma}^{(4)}|_{s=4m^2} = -\lambda$$

where the last condition is our definition of physical $\lambda$. With this choice, we have

$$\delta_m = +\frac{\lambda m^2}{32\pi^2}\left(\frac{2}{\epsilon} - \log\frac{m^2}{4\pi\mu^2} + 1 - \gamma\right)$$

to this order, and so, to this order they combine to give

$$\Pi'(p^2) = 0.$$

We also have $\delta Z = 0$ and $\delta\lambda$ is such that now

$$\tilde{\Gamma}^{(4)} = -\lambda + \frac{\lambda^2}{32\pi^2}\big(A_1(s) + A_1(t) + A_1(u) - A_1(4m^2) - 2A_1(0)\big)\,.$$

• Mention 2 loops. E.g. 5 diagrams contributing to $\Pi'(p^2)$ (one is the 2-loop counterterm, and two involve 1-loop together with 1-loop counterterms). Two cancel, with our choice above for the 1-loop diagrams. One finds e.g.

$$\delta_Z = -\frac{\lambda^2}{12(16\pi^2)^2}\frac{1}{\epsilon} + \text{finite and higher loops}$$