203a Homework 5, due March 5

1. (O'Neil) Consider two particles of charge e and mass m, initially held at $z = \pm d$ in the lab frame. The particles are then released. Call particle 1 that at $z = +d$ and particle 2 that at $z = -d$. **Keep everything fully relativistic.**

(a) Find the acceleration $\vec{a}$ of particle 1 due to the force from the fields of particle 2.

(b) Now transform to the $\prime$ rocket frame, moving with velocity $\vec{v} = v\widehat{x}$ with respect to the lab frame. Transform the fields due to charge 2 from the lab to the rocket frame, to find $\vec{E}'$ and $\vec{B}'$ in the rocket frame. Using these fields compute the force on charge 1 due to the fields of charge 2, and then solve for $\vec{a}'$ of charge 1.

(c) Compare your results from parts (a) and (b) and see if they're consistent with the relativistic formula for the transformation of accelerations.

2. Consider the following three cases for the electric and magnetic fields in some frame K (with P some constant):

i. $\vec{E} = (4P, 0, 0)$ and $\vec{B} = (0, 5P, 0)$

ii. $\vec{E} = (5P, 0, 0)$, $\vec{B} = (0, 4P, 0)$

iii. $\vec{E} = (P, 0, 0)$, $\vec{B} = (P, 2P, 0)$.

a. In which of these cases is there a frame K' where the field is purely electric? For each such case, write out the Lorentz transformation, $x^\mu = \Lambda^\mu_\nu x^{'\nu}$, between the frame K and the frame K' where $\vec{E}' = E'_0\widehat{x}$ and $\vec{B}' = 0$? What is E'_0 in terms of P?

b. In which of the above cases is there a frame K' where the field is purely magnetic? For each such case, write out the Lorentz transformation $x^\mu = \Lambda^\mu_\nu x^{'\nu}$, between the frame K and the frame K' where $\vec{E}' = 0$ and $\vec{B}' = B'_0\widehat{y}$? What is B'_0 in terms of P?

c. For the case or cases found in part (b), solve for the motion of a charge q particle which is at $x^\mu = 0$, with velocity $\vec{v} = 0$, in frame K. Solve for the trajectory $x\,'^\mu(t') = (ct', \vec{x}\,'(t'))$ seen in the frame K', where the field is purely magnetic.

3. In class we discussed an infinite wire, of constant charge per length λ' that is at rest along the $\widehat{x}$ axis in the rocket frame, that is moving with velocity $\vec{v} = v\widehat{x}$ relative to the lab frame. We found ρ and $\vec{J}$ in the lab frame.

(a) Compute $\vec{E}'$ and $\vec{B}'$ in the rocket frame and transform them to the lab frame, to find $\vec{E}$ and $\vec{B}$.

(b) Find $\vec{E}$ and $\vec{B}$ directly from the ρ and $\vec{J}$ in the lab frame. Verify that they agree with those found above.