203a Homework 4, due Feb. 6

1. In class, we verified energy conservation for a charging-up capacitor, showing that $\frac{d}{dt}U_{field} + \int_{\partial V} \vec{S}\cdot d\vec{a} = 0$. We said that we'll approximate U_{field} as coming only from the electric field inside the capacitor, neglecting the magnetic field contribution, and that this gives $U_{field,elec} \approx Q^2/2C$, where C is the capacitance, i.e. $Q = C\Delta\phi$, where ϕ is the scalar potential and $\Delta\phi$ is the voltage difference between the two plates. Suppose that you're curious as to why we neglected the $\vec{B}$ field contribution to U_{field}; then you'll enjoy verifying the following questions.

(a) Verify that $\int dVE^2/8\pi$ over the inside of the capacitor gives $U_{field,elec} = Q^2/2C$. Verify it explicitly for parallel plates of area Q, and separation d, with charge Q on one and $-Q$ on the other, neglecting end effects. Find C, both from $U_{field,elec} = Q^2/2C$ and from $Q = C\Delta\phi$ and show that they agree.

(b) Same as above, but this time for concentric cylinder plates. Again, find C the two ways and show they agree.

(c) Same as above, but for concentric spherical shell plates.

(d) Can you give a general argument for why the above always gave $U_{field} = \frac{1}{2}Q\Delta\phi$?

2. Now we suppose, as in class, that $\dot{Q} \neq 0$. The approximation in class was that the charge is changing sufficiently slowly that terms like $\dot{Q}^2$ and d^2Q/dt^2 could be neglected. Here we're going to see how things work when we go beyond that approximation. Consider two parallel disk plates, of radius R, and separation d.

(a) Find $\vec{B}$ everywhere inside the capacitor plates.

(b) Compute $U_{field,mag} \equiv \int dVB^2/8\pi$. You should find $U_{field,mag} = \frac{1}{2}L\dot{Q}^2$, where the calculation will reveal what L (the inductance of the capacitor) is. What is the approximation needed for $U_{field,elec} \gg U_{field,mag}$ to be true?

(c) Suppose that you're curious how the energy conservation equation now works. We get $\dot{U}_{field,elec} = Q\dot{Q}/C$ and $\dot{U}_{field,mag} = LI\dot{I}$, with $I \equiv \dot{Q}$. Can you show where the needed additional contribution to the energy flux, to account for $\dot{U}_{field,mag}$, comes from? Can you work it out in detail, to verify energy conservation?

3. Garg 35.2 (very similar to the above).

4. Garg 36.1.

5. Garg 36.3.

6. Garg 36.6.

7. Garg 37.1.