2/2/15 Week 5 phys 4D lecture outline

⋆ Reading: Chapters 1 and 2 of Spacetime Physics.

• Book's story about a realm where they measure distances in a perverse way. Emphasize coordinate independent concepts, lengths. Rotational symmetry in space. Vectors are geometric. Can define $d\vec{x}^2 = d\vec{x} \cdot d\vec{x}$ as geometric way to measure lengths, with $d\ell \equiv \sqrt{d\vec{x}^2}$.

• The point of the parable: it's analogously somewhat perverse to measure time and space differently. Don't put c on a pedestal. Also, don't think of c as being just the speed of light – it happens to be the speed of light, but there is more (and perhaps less) to it than that, so let's not put the horse before the cart. I'll try to emphasize this by calling c "the conversion factor" or "the speed limit". By a choice of units, we can set $c = 1$. People in my field use these units all the time. BTW, we also set $\hbar = 1$ by a choice of units, and think of $\hbar$ as a conversion factor. Examples of $c = 1$ units: measure distances in seconds (or light years); measure time in meters.

• The key idea of relativity: **no experiment can distinguish any inertial reference frame as being special; all inertial frames are physically equivalent.** No way to decide which observer is moving vs which is standing still.

Inertial frames are, as in Newton's laws, frames that are non-accelerated. Like an airplane, or rocket, that is coasting along. For later use, it is convenient to define inertial as also being without gravity. Why? Because general relativity (Einstein's later theory) shows that gravity is best thought of in a different way (spacetime curvature). Also, the apparent force of gravity has an equivalence principle replacement with acceleration. For example, standing on ground, and feeling g from the Earth's mass, could be replaced with being standing on the floor of a rocket that is accelerating at g. E.g. this shows that photons fall with acceleration g, just like a pingpong ball or bowling ball dropped off a leaning tower, even though photons have $m_\gamma = 0$. Free floating, eliminates even gravity, since everything falls at the same rate. So there's no relative acceleration in a frame that's itself falling.

• How to make a system of synchronized clocks in a reference frame. This is what "time" is: the readout of these clocks. Important idea: time itself will be frame dependent!

• The book chooses an interesting starting point. It turns out that, according to relativity, all inertial observers agree on the value of the space-time interval between two events, $ds^2 \equiv (cdt)^2 - d\vec{x}^2$. Let's call the two inertial observers A and B (e.g. Alice and Bob). So A measures dt_A and $d\vec{x}_A$ and B measures dt_B and $d\vec{x}_B$ as the time and space coordinate differences between the two events. All these coordinate quantities generally differ, but the space-time geometric quantity ds^2 is invariant: $ds_A^2 = ds_B^2$. We will understand why this is true better later. Following the book, we'll just state this as something that you'll accept for the moment, and then we'll study the implications.

Write also $\Delta s^2 \equiv c\Delta t^2 - \Delta\vec{x}^2$.

• Example: event 1: flash of light created; event 2: light detected by your eye. Two events separated by $\Delta s^2 = 0$ in any inertial frame.

• Aside on mostly minus vs mostly plus conventions for Δs^2. It's all just a matter of convention, no significance to overall sign, just the relative minus sign of space vs time matters. The reason for that relative minus sign is a deep mystery. It's good to stick with a convention, and we'll stick with mostly minus convention. The book suggests instead to define ds^2 as $|cdt^2 - d\vec{x}^2|$, which is not standard and I do not endorse that – I definitely recommend against it. Following usual conventions, with the mostly minus choice, we just have $ds^2 > 0$ for two events that are time-like separated, $ds^2 < 0$ for two events that are spacelike separated, and $ds^2 = 0$ for two events that are light-like (aka null) separated.

• Proper, or wristwatch time: for two events that are time-like separated, $\Delta s^2 \equiv (c\Delta\tau)^2$, where τ is the wristwatch time interval, i.e. the time between the two events in the frame of reference where they occur at the same point in spacetime.

• Measure velocities in units of c, $\beta \equiv v/c$. In $c = 1$ units, $\beta = v$.

• Example: suppose a rocket moves with $\beta = 99/100$ and we see two events: the rocket's dinger (attached to the rocket) dings two bells (attached to strings in the lab). Event 1 in the lab is at $t = 0$, $\vec{x} = 0$. Event 2 is at $ct = 100m$, $x = 99m$. What is the time separation of the two events in the rocket frame? Compute $\Delta s^2 = (100m)^2 - (99m)^2 = c\Delta\tau^2$. So the time separation in the rocket frame, where the two events occur at the same location (that of the rocket's dinger), is $c\Delta\tau = \sqrt{(100)^2 - (99)^2}m$.

• Continue from my handwritten notes: spacetime diagrams and world lines; time dilation; conflict with simultaneity between frames; synchronized lattice of clocks and times between frames; time dilation and length contraction to explain muons hitting ground, depending on frame; no transverse length contraction; light clocks in directions parallel and perpendicular to the motion, used to get time dilation and length contraction again.