3/1/16 Lecture 17 outline

• Last time: Recall spin 1 gauge field canonical quantization, and A_μ as an operator. Recall gauge invariance of $\mathcal{L}_{EM}$, needed to avoid having wrong sign kinetic term for longitudinal polarization terms.

• Functional integral for gauge fields. Important point: gauge invariance. Write $A = A_\mu dx^\mu$. Recall gauge symmetry $A \to A^\alpha = A + d\alpha(x)$, with $\psi_i \to e^{-iq_i\alpha(x)}\psi$. Redundancy in description, can only observe gauge invariant quantities. Need to replace $\partial_\mu\psi_i \to D_\mu\psi_i \equiv (\partial_\mu + iq_iA_\mu)\psi_i$. Then $D^\alpha_\mu\psi^\alpha_i = e^{-iq_i\alpha}D_\mu\psi_i$ transforms nicely, with just an overall phase, and $\bar\psi_i D_\mu\psi_i$ is gauge invariant. So the Dirac lagrangian, $\bar\psi(i\not{D} - m)\psi$ is gauge invariant.

The terms linear in A_μ give $\mathcal{L} \supset -A_\mu j^\mu$, with j^μ the conserved current.

• In the functional integral, will have $\int[dA]\exp(iS)$. Integration measure must be gauge invariant, implies it gets a factor of gauge orbit volume. Would like to integrate only over a slice of inequivalent gauge fields, without integrating over the gauge orbits. Need to do this, since otherwise there is no well defined B^{-1}. Recall $S = \int d^4x[-\frac{1}{4}F^2_{\mu\nu}] = \frac{1}{2}\int d^4kA_\mu(x)(\partial^2 g^{\mu\nu} - \partial^\mu\partial^\nu)A_\nu(x)$. Note action vanishes if $\tilde A_\mu(k) = k_\mu\alpha(k)$. Gauge invariance. $A^T_\mu = P_{\mu\nu}A^\nu$, $P_{\mu\nu} = g_{\mu\nu} - \partial_\mu\partial_\nu/\partial^2$. $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}A^T_\mu\partial^2 g^{\mu\nu}A^T_\nu$. Can't invert kinetic terms uniquely to find Green's function. We need to fix the gauge.

The functional integral should be over $\int[dA^\mu]/(GE)$, where we divide by the volume of the gauge equivalent orbits. The gauge equivalent orbits are associated with gauge transformations $\alpha(x)$, e.g. $A_\mu \to A_\mu + \partial_\mu\alpha(x)$ in the Abelian case. We want to do the functional integral over A^μ, dividing out by the $\alpha(x)$.

(Here are some details: Do this via

$$1 = \int[d\alpha(x)]\delta(G(A^\alpha))\det\left(\frac{\delta G(A^\alpha)}{\delta\alpha}\right) = \Delta\int[d\alpha]\delta(G(A^\alpha)),$$

where $G(A) = 0$ is some gauge fixing condition, e.g. Lorentz gauge, $G(A) = \partial_\mu A^\mu$ and

$$\Delta = \det\left(\frac{\delta G(A^\alpha)}{\delta\alpha}\right)_{G=0}.$$

Δ is the Faddeev-Popov determinant. Write the functional integral as (using the gauge invariance of measure and action)

$$\int[d\alpha][dA]\Delta\delta(G[A])\exp(iS[A]).$$

Have factored out the integral over the group volume. We can then just easily divide out by $[d\alpha]$, just cross it out. What's left is the gauge fixing delta function, and appropriate determinant factor.