2/16/18 Physics 212b Homework 4, due Monday Feb 26

1. A one-dimensional harmonic oscillator of charge e is perturbed by an electric field of strength ϵ in the $\hat{z}$ direction. Calculate the change in each energy level to second order in the perturbation, and calculate the induced electric dipole moment. Show that this problem can be solved exactly and compare the result with the perturbation approximation.

2. Repeat the previous exercise for the 3d isotropic harmonic oscillator. Show that, if the polarizability α of the oscillator is defined as the ratio of the induced electric dipole moment to ϵ, the change in energy is exactly $-\frac{1}{2}\alpha\epsilon^2$.

3. A one-dimensional harmonic oscillator is perturbed by an extra potential energy ϵx^3. Calculate the change in each energy level to second order in the perturbation.

4. Consider the hydrogen atom and model the proton as a uniformly charged sphere of radius $r_p \ll a_0$, treating the electron as a point charge in the associated potential $\phi(r)$. Treat this as a perturbation, and write down the associated $H^1(r)$. Compute the change in energy of the ground state to first order in H^1.

5. A one-electron atom whose ground state is non-degenerate is placed in a uniform electric field in the $\hat{z}$ direction. Obtain an approximate expression for the induced electric dipole moment of the ground state by considering the expectation value of ez with respect to the perturbed eigenstate to first order. Show that the same expression can be obtained from the energy shift $-\frac{1}{2}\alpha|\vec{E}|^2$ of the ground state computed to second order, where α is the polarizability.

6. A system with three states has Hamiltonian:

$$H = \begin{pmatrix} E_1 & 0 & a \\ 0 & E_1 & b \\ a^* & b^* & E_2 \end{pmatrix}$$

where $a \sim b \sim \epsilon$, with $\epsilon \ll E_1 < E_2$. Use second-order non-degenerate perturbation theory to calculate the perturbed eigenvalues (is this procedure correct?). Then diagonalize the matrix to find the exact eigenvalues. Finally use second-order degenerate perturbation theory. Compare the three sets of results.