3/5/18 Physics 212b Homework 5, due Friday Mar 16. Please turn in to **Sridip**

1. Estimate the groundstate energy of a 3d particle of mass m with $V = -Ze^2/r$ by taking $\psi_{trial}(r) = Ae^{-\lambda r^2}$. Solve for λ and the minimum E_{trial} and verify $E_{trial} > E_{actual}$.

2. Estimate the groundstate energy of a 3d particle of mass m, with $V = -Ze^{-qr}/r$ (where Z and q are some non-negative constants), by taking $\psi_{trial} = Ae^{-\lambda r}$. You can use $\Gamma[z] = \int_0^\infty dx x^{z-1} e^{-x}$ if you like.

3. The groundstate of a hydrogen atom is subject to a potential $V = V_0 \cos(kz - \omega t)$. Using time-dependent perturbation theory obtain (to leading order) an expression for the transition rate at which the electron is ionized and emitted with momentum $\vec{p}$. Show how to compute the angular distribution in terms of θ and ϕ, defined with respect to the z axis. You can take the final state to be $\psi_f(\vec{x}) = e^{i\vec{p}\cdot\vec{x}/\hbar}/L^{3/2}$, and note that observable effects are independent of L.

4. Consider a 1d SHO: $H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega_0 x^2$ and perturb by $H_1 = \Theta(t)F_0 x \cos\omega t$, where F_0 is a constant in space and time. Suppose that $|\psi(t < 0)\rangle = |0\rangle$. Obtain an expression for $\langle x\rangle$ as a function of time to lowest non-vanishing order in time-dependent perturbation theory. Is this procedure valid for $\omega = \omega_0$?

5. Consider a particle in a 1d SHO potential, which is in the groundstate for $t < 0$. For $t > 0$ the perturbation $H_1 = \Theta(t)Ax^2 e^{-t/\tau}$ is turned on. Using time dependent perturbation theory, compute the probability that at late times $t \gg \tau$, the system has made a transition to a given excited state (consider all final states).

6. A hydrogen atom is in its groundstate and is perturbed by the electric field $\vec{E} = E_0\hat{z}\Theta(t)e^{-t/\tau}$. Using time dependent, first-order perturbation theory, compute the probability for the atom at late times to be found in each of the three $2p$ states, as well as the $2s$ state. You do not need to evaluate the radial integrals if you don't want to, but please do all other integrations (angles and time).

7. Consider an atom made up of an electron and a singly charge ($Z = 1$) triton (yay!, it's a ^3H nucleus consisting of two neutrons and a proton). The system is initially in the groundstate, and then the triton suddenly beta decays to a $Z = 2$ nucleus (a neutron decays to a proton, electron, and anti-neutrino). What is the probability of finding the electron in the groundstate of the resulting He ion.

8. Consider a system of two distinguishable spin 1/2 objects, with $H = \Theta(t)(4\Delta/\hbar^2)\vec{S}_1 \cdot \vec{S}_2$. Suppose that the system is in the state $|+-\rangle$ for $t \leq 0$. Find the probability as a function of time for its being in each of the four possible $|\pm\pm\rangle$ states

(a) By solving the problem exactly.

(b) By solving the problem using first order perturbation theory, treating H as a perturbation. Under what condition does this give the correct results?