110b HW Due 3/11/20

1. As discussed in lecture, $H = \sqrt{(c\vec{p} - q\vec{A}(t,\vec{x}))^2 + (mc^2)^2} + q\phi(t,\vec{x})$.

   (a) Use Hamilton's equations to obtain $\vec{v} = \frac{d\vec{x}}{dt}$. Use this to show

$$\sqrt{(c\vec{p} - q\vec{A})^2 + (mc^2)^2} = \gamma mc^2, \qquad \text{and} \qquad \vec{p} = \gamma m\vec{v} + \frac{q}{c}\vec{A}.$$

   (b) Use Hamilton's equation to obtain $\frac{dp^i}{dt}$ (hint: write all the vector indices out, rather than using vector and dot product notation, to avoid making errors). Show that this leads to the Lorentz force law in the form

$$\frac{d}{dt}(\gamma m\vec{v}) = q(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}) \quad \text{so} \quad \vec{f} = m\frac{d\vec{u}}{d\tau} = \frac{q}{c}(\vec{E}u^0 + \vec{u} \times \vec{B}).$$

2. Verify that the Lorentz transformation of $F^{\mu\nu}$ (for the case of a boost along the x axis) gives the transformations of $\vec{E}$ and $\vec{B}$ that were discussed in a lecture.

3. Suppose that $T^{\mu\nu} = (pc^{-2} + \rho)u^\mu u^\nu - p\eta^{\mu\nu}$ where ρ and p are Lorentz invariants and u^μ is the 4-velocity of the frame relative to the rest frame of a fluid that fills the space.

   (a) Compute $T^{\mu\nu}$ in the rest frame of the fluid.

   (b) Compute $T^{\mu'\nu'}$ in a frame that is moving with velocity $\vec{v} = \frac{4}{5}c\hat{x}$ relative to the rest frame.

   (c) Compute $\eta_{\mu\nu}T^{\mu\nu}$ in both frames. Do they agree? Should they?

4. (a) Taylor 16.24.

   (b) Taylor 16.34.