215a Homework exercises 4, Fall 2015, due Nov. 4

1. Consider the theory $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \rho(x)\phi$, where $\rho(x^\mu)$ is an external source. [This exercise is taken from Peskin and Schroeder 4.1. You can find much of this worked out in the Coleman lecture notes, but please try to think about it and not peek / copy too much.]

(a) Give some words / explanation that the probability that the source creates no particles is given by

$$P(0) = |\langle 0|Te^{i\int d^4x\rho\phi_I}|0\rangle|^2,$$

where ϕ_I means interaction picture.

(b) Evaluate the above $P(0)$ to order ρ^2, thinking of ρ as a small perturbation. Show that $P(0) = 1 - \lambda + \mathcal{O}(\rho^4)$, where

$$\lambda \equiv \int \frac{d^3p}{(2\pi)^3(2E_p)}|\widetilde{\rho}(p)|^2,$$

where $\widetilde{\rho}(p) \equiv \int d^4x e^{ipx}\rho(x)$ (a tilde is often used to remind that it's a Fourier transform).

(c) Represent the term contributed in part (b) as a Feynman diagram. Now represent the whole perturbation series for $P(0)$ in terms of Feynman diagrams. Show that the series exponentiates so that it can be summed exactly: $P(0) = e^{-\lambda}$.

(d) Compute the probability that the source creates one particle of momentum k. Perform this first to $\mathcal{O}(\rho)$ and then exactly, using the trick of part (c).

(e) Show that the probability of producing n particles is given by the Poisson distribution:

$$P(n) = \frac{\lambda^n}{n!}e^{-\lambda}.$$

(f) Note that $\sum_{n=0}^{\infty} P(n) = 1$ and $\langle N\rangle_{ave} \equiv \sum_{n=0}^{\infty} nP(n) = \lambda$, so λ is the expected number of created particles. Compute the mean square fluctuation $\langle (N - \langle N\rangle_{ave})^2\rangle_{ave}$.

2. Suppose that we modified our nucleon + meson model by adding $\Delta\mathcal{L}_{int} = g'\phi\psi\psi + g'\phi\psi^\dagger\psi^\dagger$, and do a perturbative expansion in both g and g'.

(a) Draw the new Feynman diagram interaction vertices associated with g'. As in class, use dotted lines to represent ϕ and directed arrow lines to represent ψ and $\psi^\dagger$. The g' interaction violates the previously conserved nucleon number, so the associated vertices can act as a net source or sink for arrows.

(b) Consider $N(p_1) + N(p_2) \to N(q_1) + N(q_2)$. Show that $g' \neq 0$ allows for an s-channel diagram. Draw the diagram and give its contribution to the scattering amplitude, to leading non-zero order.